Social trust-driven consensus reaching model with a minimum adjustment feedback mechanism considering assessments-modifications willingness

Hengjie Zhang, Fang Wang, Yucheng Dong, Francisco Chiclana, Enrique Herrera-Viedma

Abstract—Social network group decision-making (SNGDM) has emerged as a new decision tool to effectively model the social trust relationships among decision makers. The impact of the social trust relationships on assessments-modifications in the consensus reaching in the SNGDM is seldom considered. This study aims at addressing this issue. The main starting point is the assumption that a decision maker will not be willing to accept the assessments-modifications suggestions that significantly differ from his/her trusted decision makers’ assessments in a social trust network. Thus, this study proposes a social trust-driven minimum adjustments consensus model (STDMACM) for SNGDM. Simultaneously, a social trust-driven consensus maximum optimization model (STDCMOM) is proposed for maximizing the consensus level among decision makers under the above assumption. Based on both STDCMOM and STDMACM, an interactive consensus reaching process is presented, in which the assessments-modifications suggestions generated from the STDMACM are used, when the maximum consensus level obtained from STDCMOM is acceptable, as the references for guiding the consensus reaching; otherwise, assessments-modifications suggestions are generated from the designed STDCMOM. The validity of the social trust-driven consensus reaching process with respect to its consensus convergence rate and consensus success ratio is verified with a simulation and comparison analysis.

Index Terms — Social network group decision making, consensus, assessments-modifications, minimum adjustment, optimization model

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I. INTRODUCTION

Group decision making (GDM) is a powerful tool to settle complex decision problems [1, 2], which can overcome the predicament that a single decision maker may not be able to consider all aspects of a decision problem. Numerous GDM methods have been designed to integrate the levels of knowledge and experience associated with multiple decision makers (e.g., [3-8]). Traditional GDM methods mainly focused on obtaining the preference ordering of alternatives and failed to solve the issue of assessment divergence. For this reason, the consensus model has been designed and embedded into GDM for improving the consensus level among decision makers regarding their assessments [9-15].

Traditionally, “hard” consensus measures were proposed in the GDM to sustain the consensus building. Nevertheless, achieving a complete consensus may be resource-consuming and impractical in many decision-making problems [16]. Accordingly, the concept of “soft” consensus measure was devised to overcome the shortcomings of the “hard” consensus measure concept [16]. Moreover, decision makers often need to modify their assessments in order to build a consensus in the GDM [6]. Notably, assessments-modifications mean cost and the resources for consensus building are often limited in many cases. Consequently, a great deal of consensus models with minimum adjustments (or cost) have been devised for resource saving (e.g., [17-23]). For example, Labella et al. [24] proposed a cost consensus metric for consensus reaching process based on a comprehensive minimum cost model; Xu et al. [25] studied the impact of decision rules and non-cooperative behaviors on minimum consensus cost in GDM; Wu et al. [26] developed several multi-stage optimization models with minimum adjustments for managing individual consistency and group consensus with preference relations; while Cheng et al. [27] proposed several minimum cost consensus models based on the concept of asymmetric unit costs. Gong et al. [28] investigated consistency and consensus issues in GDM with linear uncertain preference relations. More detailed information about minimum adjustments (or cost) consensus models will be provided in Section II.B.

Recently, the development of information technology and society promotes the emergence of social network group decision making (SNGDM) methods (e.g., [29-31]). Particularly, several consensus models have been devised for SNGDM problems (e.g., [32, 33]). Ureña et al. [34] proposed a similarity based influence social network to reach consensus among a group decision makers involved in the SNGDM process; Wu et al. [35] designed a visual interaction consensus model for SNGDM with trust propagation; Ding et al. [36] proposed a social network analysis-based conflict relationship investigation and conflict degree-based consensus model for large-scale SNGDM using sparse representation; while Zhang et al. [37] proposed an approach to manage non-cooperative behaviors in the consensus-based SNGDM. Additional consensus models for SNGDM can be found in Ref. [29].
Although many studies have discussed consensus models in the SNGDM framework, there are still some issues that need to be addressed.

(1) Limited work has been carried out on modeling minimum adjustments for reaching consensus in SNGDM. The quantification of information loss is a key criterion to assess the performance of a consensus model [38]. However, this has rarely been considered in existing SNGDM consensus models. Thus, it is necessary to study how consensus model with minimum adjustments (or cost) can be implemented in SNGDM to improve the efficiency of the consensus reaching process.

(2) In consensus-based SNGDM, the impact of social trust relationships on the assessments-modifications is not fully considered. In SNGDM, decision makers are assumed to be influenced by their trusted decision makers with regard to assessments modification. This phenomenon is universal and has been investigated and studied by some scholars in the opinion dynamics discipline (e.g., [39, 40]). To the best of our knowledge, the influence of social trust relationships on assessments-modifications in the feedback mechanism is not fully considered in existing consensus-based SNGDM methods. Hence, it is worth investigating the impact of social trust relationships in the SNGDM consensus reaching process.

Motivated by the challenge to overcome the issues of the consensus-based SNGDM methods analyzed above, this study develops a social trust-driven minimum adjustments consensus model (STDMACM) for SNGDM that takes into account the impact of the social trust relationships on the assessments-modifications. Notably, the STDMACM is based on the aforementioned assumption that a decision maker will accept the assessments-modifications suggestions they receive when those suggestions are close to his/her trusted decision makers’ assessments in the considered social trust network. Before applying the STDMACM, a social trust-driven consensus maximum optimization model (STDCMOM) is carried out, which aims at maximizing the group consensus level by seeking their optimal adjusted assessments driven by the above assumption. If the obtained maximum group consensus level is acceptable, then the STDMACM is adopted to generate assessments-modifications suggestions; otherwise, the assessments-modifications suggestions yielded from the application of STDCMOM are used as references for promoting the consensus reaching. Moreover, a simulation and comparison analysis are devised to discuss the validity of the proposal, put forwards herein, whose results confirm that consensus efficiency in terms of consensus convergence rate and consensus success ratio improves when the impact of social trust relationships on the assessments-modifications is taken into account.

The rest of this research paper is organized as follows. Section II offers some basic knowledge on social trust network structure and minimum adjustments (or cost) consensus model to make this study self-contained. Section III describes the SNGDM problem and puts forward its resolution framework and procedures. Following this, an illustrate example is given in Section IV. Next, detailed simulation and comparison analysis are conducted in Section V. Finally, Section VI concludes the study and discusses the research directions for the future.

II. PRELIMINARY KNOWLEDGE

In order to help the understanding of later parts of this paper, this section provides the basic knowledge about the structure of social trust network and the minimum adjustments (or cost) consensus model.

A. The structure of social trust network

Let $E = \{e_1, e_2, ..., e_m\}$ be the set of $m$ individuals (or decision makers). In general, the principal elements involved in a social trust network are described by the following two representation schemes [41, 42]:

(1) Graph theoretic: a graph is used to characterize the trust relationship among individuals. Concretely, in the graph, $e_i \rightarrow e_j$ denotes that there is a direct trust from individual $e_i$ to individual $e_j$.

(2) Sociometric: in this scheme, the trust relationships among individuals is represented with a matrix $T = (t_{ij})_{mn}$, where $t_{ij} = 1$ signifies that there exists a direct trust from individual $e_i$ to individual $e_j$, while $t_{ij} = 0$ represents that there is no direct trust relationship from individual $e_i$ to individual $e_j$. The matrix $T = (t_{ij})_{mn}$ is called sociomatrix.

In Table 1, we provide an example to show the different representation schemes for a social trust network.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Sociometric</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>$e_2$</td>
</tr>
<tr>
<td><img src="image-url" alt="Graph Example" /></td>
<td>$T = \begin{bmatrix} 0 &amp; 0 &amp; 1 &amp; 1 \ 1 &amp; 0 &amp; 1 &amp; 1 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 1 &amp; 0 &amp; 1 &amp; 0 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

It is worth mentioning that the above binary sociomatrix cannot fully depict the trust relationship between individuals [43], which is done by using the extended concept of fuzzy sociomatrix. Specifically, in a fuzzy sociomatrix $T = (t_{ij})_{mn}$, its element $t_{ij} \in [0, 1]$ represents the trust value that individual $e_i$ distributes to individual $e_j$. As there is no risk of confusion, a fuzzy sociomatrix will also be referred to as simply sociomatrix herein.

B. Minimum adjustments (or cost) consensus model

The origin and different variants of the minimum adjustments (or cost) consensus model are described in this section.

As aforementioned, decision makers are often required to modify their assessments in order to reach consensus in the GDM. Let the original and adjusted assessments provided by the decision makers $E = \{e_1, e_2, ..., e_m\}$ be denoted by $\{p_1, p_2, ..., p_m\}$ and $\{\bar{p}_1, \bar{p}_2, ..., \bar{p}_m\}$, respectively. To keep the original assessments of decision makers as much as possible, Dong et al. [18, 45] designed a minimum adjustments consensus model within a linguistic framework, which herein is presented in its numerical form with decision variables $\{\bar{p}_1, \bar{p}_2, ..., \bar{p}_m\}$:

$$\min \sum_{k=1}^{m} d(p^k, \bar{p}^k)$$

$$s.t. \begin{bmatrix} \bar{p} = F(\bar{p}^1, \bar{p}^2, ..., \bar{p}^m) \\ CL(\bar{p}^k, \bar{p}^k) \geq \alpha, k = 1, ..., m \end{bmatrix}$$ (1)

In model (1), $d(p^k, \bar{p}^k)$ measures the distance between the original assessment $p^k$ and its corresponding adjusted assessment $\bar{p}^k$. The objective of model (1) is to minimize the distance between $\{p_1, p_2, ..., p_m\}$ and $\{\bar{p}_1, \bar{p}_2, ..., \bar{p}_m\}$. The first constraint aims to obtain adjusted collective assessment $\bar{p}$ from $\{\bar{p}_1, \bar{p}_2, ..., \bar{p}_m\}$ using aggregation function $F$. The second constraint guarantees that the consensus levels of all decision makers are acceptable, where $\alpha$ is the predefined consensus level.
Meanwhile, Ben-Arie and Easton [17] designed a consensus reaching algorithm that considered consensus cost but no specific optimization model was involved, which was subsequently addressed by Zhang et al. [19] with the designing of an optimization-based minimum cost consensus model, which was proved to be connected with Dong et al.’s minimum adjustment consensus model [18] via a general optimization-based consensus model. To date, a great deal of consensus models with minimum adjustments (or cost) have been devised and used in various GDM contexts [17-19, 44]. Dong and Xu [45] and Zhang et al. [21] provided a comprehensive review regarding the minimum adjustments (or cost) consensus models, and the impact of social trust relationships on the assessments-modifications stage of the GDM consensus reaching process, as aimed in this study, is not being accounted for.

III. SOCIAL NETWORK GROUP DECISION MAKING PROBLEM AND ITS RESOLUTION FRAMEWORK AND PROCEDURES

In this section, the SNGDM problem is formally presented and a framework with detailed procedures to facilitate its resolution is designed.

A. The social network group decision making problem

Let \( M = \{1,2,...,m\} \), \( N = \{1,2,...,n\} \), and \( H = \{1,2,...,h\} \). Recall that \( E = \{e_1, e_2, ..., e_n\} \) is the set of \( n \) \((n \geq 2)\) decision makers. The vector \( \lambda = (\lambda_1, \lambda_2, ..., \lambda_m)^T \) represents decision makers’ importance weights: \( \lambda_k \in [0,1] \) and \( \sum_{k=1}^{m} \lambda_k = 1 \). It is assumed that there exist trust relationships among the decision makers, which is represented by a sociomatrix \( T = (t_{ij})_{m \times n} \). In the decision process, there are \( n \) \((n \geq 2)\) feasible alternatives \( X = \{x_1, x_2, ..., x_n\} \), which are evaluated with respect to a set of \( h \) \((h \geq 2)\) attributes \( B = \{b_1, b_2, ..., b_h\} \). The weighting vector over \( B \) is \( \beta = (\beta_1, \beta_2, ..., \beta_h)^T \), where \( \beta_i \in [0,1] \) and \( \sum_{i=1}^{h} \beta_i = 1 \). Each decision maker \( e_k \) \((k \in M)\) provides a multiple attribute assessment matrix (MAAM) \( A_k = (a_{ij}^k)_{m \times n} \), where element \( a_{ij}^k \) denotes its assessment value of alternative \( x_i \) with respect to attribute \( b_j \). Without loss of generality, in this study, it is assumed that \( a_{ij}^k \in [0,1] \) \((k \in M; i \in N; j \in H)\).

The aim of any SNGDM problem resolution procedure is to obtain a consensual collective MAAM \( A^* = (a_{ij}^*)_{m \times n} \), so as to yield a final consensual preference ordering of alternatives.

B. Resolution framework and procedures for the SNGDM problem

In general, a SNGDM resolution procedure includes two processes[46]: the consensus reaching process and the selection process. The first process is used to help decision makers reach an acceptable level of consensus while the second process leads to finding the best alternative or to producing a ranking of the alternatives. As aforementioned, this study aims at designing a social trust-driven consensus reaching framework with minimum adjustments for the above SNGDM problem. As illustrated in Fig. 1, the proposed social trust-driven consensus reaching framework involves the following key procedures:

**Step 1: Social network analysis**
At this step, detailed in Section B.1, the decision makers’ weights are generated and the social trust based feedback adjustment mechanism driven by the social trust relationships among decision makers is designed.

**Step 2: Application of social trust driven consensus model with minimum adjustments to generate assessments-modifications suggestions**
At this step, elaborated in Section B.2, two social trust driven consensus models (STDCMOM and STDMACM) to generate assessments-modifications to help decision makers reach a consensus regarding collective MAAM are presented.

**Step 3: Selection process**
At this step, the preference ordering of alternatives is generated from the obtained collective MAAM at Step 2.
It is well known that there are two categories of attributes in a multiple attribute decision making: benefit and cost attributes. Based on these two categories of attributes, a MAAM \( A^* = (a_{ij}^*)_{m \times n} \) is transformed into its corresponding normalized MAAM \( V^* = (v_{ij}^*)_{m \times n} \) [47] where:

\[
v_{ij}^* = \frac{a_{ij}^* - \min_{i \in N} a_{ij}^*}{\max_{i \in N} a_{ij}^* - \min_{i \in N} a_{ij}^*} \quad \text{if} \quad b_j \in B \quad \text{is a benefit attribute}
\]

while

\[
v_{ij}^* = \frac{\max_{i \in N} a_{ij}^* - a_{ij}^*}{\max_{i \in N} a_{ij}^* - \min_{i \in N} a_{ij}^*} \quad \text{if} \quad b_j \in B \quad \text{is a cost attribute}
\]

The total assessment value associated with alternative \( x_i \) is:

\[
p_{ij}^* = \sum_{j=1}^{h} \beta_j \cdot v_{ij}^*
\]

The preference ordering of alternatives derived from \( p = (p_{1j}, p_{2j}, ..., p_{nj})^T \) is represented by \( o^j = (o_{1j}^*, o_{2j}^*, ..., o_{nj}^*)^T \), where

\[
o_{ij}^* = j
\]

if \( p_{ij}^* \) is the \( j \)th largest value in \( \{p_{11}^*, p_{21}^*, ..., p_{nj}^*\} \).
B.1. Social trust network analysis

The decision makers’ weights and the social trust based feedback adjustment mechanism driven by the social trust relationships among decision makers are generated at this step.  

(1) Determining the decision makers’ weights

From the sociomatrix $T$, the relative node in-degree centrality index of each decision maker $e_i \in E$ is computed

$$IC^i = \frac{1}{m-1} \sum_{j \in v^i} t_{ij}$$  

(6)

In the SNGDM, the decision makers’ weights are generated from $\{IC^1, IC^2, ..., IC^m\}$ based on the principle that a decision maker with a high relative node in-degree centrality index should be allocated a high importance degree[29]. Following this basic idea, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)^T$ is determined as follow:

$$\lambda_k = \frac{IC^k}{\sum_{i=1}^{m} IC^i}$$  

(7)

(2) Social trust feedback adjustment

When updating assessments in the SNGDM, decision makers will have, on one hand, confidence regarding their current assessments; while, on the other hand, they will refer to the assessments of their trusted decision makers. These two assumptions are considered in designing the below social trust driven feedback adjustment.

Let $TE^k$ be the set of decision makers trusted by decision maker $e_i$ in the social trust network. Based on the network sociomatrix, the following value

$$w^i_j = \begin{cases} \frac{t_{ij}}{\sum_{j \in v^i} t_{ij}}, & \text{if } e_j \in TE^i \\ 0, & \text{if } e_j \notin TE^i \end{cases}$$  

(8)

can be interpreted as the weight value that decision maker $e_j$ assigns to the decision maker $e_i$.

Let $\theta \in [0,1]$ be the self-confidence level that decision maker $e_i$ ($k \in M$) gives to his/her own assessment, while $(1-\theta)$ is his/her self-confidence level over the assessments of his/her trusted decision makers $TE^i$. Inspired by the opinion evolution model [40], decision maker $e_i$ ($k \in M$) will accept the following benchmark MAAM $\hat{A}^i = (\hat{a}^i_{jk})_{n,n}$ when updating his/her MAAM $A^i$:

$$\hat{a}^i_{jk} = \theta \cdot a^i_{jk} + (1-\theta) \sum_{j \in v^i} w^i_j \cdot a'_j$$  

(9)

Thus, MAAM $\hat{A}^i$ can be used as a reference for decision maker $e_i$ ($k \in M$) to modify his/her MAAM $A^i$ in the feedback adjustment process of the SNGDM resolution procedure. We refer to this consensus model as the social trust driven consensus model (STDCM).

B.2. Social trust driven consensus model with minimum adjustments

A method for measuring the consensus level among decision makers is presented, and two social trust driven consensus models to help decision makers to generate assessments-modifications suggestions are designed.

(1) Consensus measure

(i) Using the decision makers’ weights the collective MAAM $A^* = (a^*_{ij})_{n,n}$ is computed as the following weighted average of the set of individual MAAMs $\{A^i, A^2, ..., A^m\}$:

$$a^*_{ij} = \sum_{k=1}^{m} \lambda_k \cdot a^k_{ij} \quad (i \in N, j \in H)$$  

(10)

(ii) The consensus level of decision maker $e_i$ regarding his/her MAAM $A^i$ is computed as:

$$CL(A^i) = 1 - \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{n} |a^i_{ij} - a^*_ij|$$  

(11)

(iii) The consensus level among all decision makers regarding their MAAMs $\{A^1, A^2, ..., A^m\}$ is:

$$CL(A^1, A^2, ..., A^m) = 1 - \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{n} |a^i_{ij} - a^*_ij|$$  

(12)

We notice that $\text{CL}(A^1, A^2, ..., A^m) \in [0,1]$. The closer the value $\text{CL}(A^1, A^2, ..., A^m)$ is to 1, the closer will be all decision makers’ assessments to the corresponding collective assessments. Therefore, $\text{CL}(A^1, A^2, ..., A^m)$ measures agreement of decision makers in terms of the closeness of their assessments. In any consensus reaching process, a minimum consensus threshold $\alpha \in [0,1]$ is often set for the group of decision makers to be satisfied before proceeding to selecting their final solution of consensus, i.e. consensus among decision makers is achieved only when $\text{CL}(A^1, A^2, ..., A^m) \geq \alpha$; otherwise, the decision makers need to modify their MAAMs to facilitate the reaching of the threshold of consensus.

(2) Two social trust driven consensus models

Using Eq. (9) to provide assessments-modifications suggestions will, on one hand, guarantee that decision makers will accept them but, on the other hand, their associated cost (information loss) may be greater than required. To overcome this, a simultaneous minimization of information loss approach with the use of Eq. (9) to keep the decision makers’ willingness to accept the assessments-modifications suggestions is proposed.

Let $\{\tilde{A^1}, ..., \tilde{A^n}\}$ denote the adjusted MAAMs associated with $\{A^1, ..., A^n\}$. Several approaches can be conducted to measure the deviation between $\{\tilde{A^1}, ..., \tilde{A^n}\}$ and $\{A^1, ..., A^n\}$, such as Manhattan distance, Euclidean distance and Minkowski distance. Without loss of generality, Manhattan distance-based approach is used in this study. To measure the smaller the deviation between $\{\tilde{A^1}, ..., \tilde{A^n}\}$ and $\{A^1, ..., A^n\}$, the higher consensus efficiency is. Thus, the aim is to minimize the distance between $\{A^1, ..., A^n\}$ and $\{\tilde{A^1}, ..., \tilde{A^n}\}$:

$$\min \frac{1}{m \cdot n} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{n} |a^i_{jk} - \tilde{a}^i_{jk}|$$  

(13)

In addition, whether the decision maker $e_i$ ($k \in M$) will accept the MAAM $\tilde{A^i} = (\tilde{a}^i_{jk})_{n,n}$ or not is to be taken into account as well. Accordingly, we present the following assumptions:

Assumption 1: Decision maker will accept the assessment-modification suggestion $\tilde{a}^i_{jk}$ if it is in a neighborhood of $a^i_{jk}$, i.e.

$$|\tilde{a}^i_{jk} - a^i_{jk}| \leq \varepsilon_k \quad (k \in M; \ v \in N; j \in H)$$  

(14)

where $\varepsilon_k \in [0,1] (k \in M)$ is the tolerance degree of decision maker $e_i$ for each round of feedback adjustment process.

Assumption 2: The consensus level among MAAMs $\{\tilde{A^1}, ..., \tilde{A^n}\}$ should be acceptable, i.e.

$$\text{CCL}(\tilde{A^1}, ..., \tilde{A^n}) = 1 - \frac{1}{m \cdot n} \sum_{i=1}^{m} \sum_{j=1}^{n} |\tilde{a}^i_{jk} - a^i_{jk}| \geq \alpha$$  

(15)
where $\hat{a}_{ij}^c = \sum_{k=1}^{m} \alpha_k \cdot \hat{a}_{ij}^k$ ($i \in N; j \in H$).

Based on the above analysis, the following STDMACM non-linear programming model, with decision variables $(\bar{A} = (\hat{a}_{ij}^c)_{mn} ,... , \bar{A}^n = (\hat{a}_{ij}^c)_{mn}, \bar{A} = (\hat{a}_{ij}^c)_{mn})$ which is referred to as model $P_1$, is introduced:

\[
\min \frac{1}{mnh} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} \left[ a_{ij}^k - \hat{a}_{ij}^c \right] \]

subject to

\[
CCL(\bar{A}, ..., \bar{A}^n) = 1 - \frac{1}{mnh} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} \left[ a_{ij}^k - \hat{a}_{ij}^c \right] \geq \alpha \quad (a)
\]

\[
\hat{a}_{ij}^c = \sum_{k=1}^{m} \alpha_k \cdot \hat{a}_{ij}^k , \quad \forall i \in N; j \in H \quad (b)
\]

\[
\sum_{k=1}^{m} \hat{a}_{ij}^c \geq \alpha \quad (c)
\]

\[
\hat{a}_{ij}^c \leq d_{ij}^c , \quad \forall i \in N; j \in H \quad (d)
\]

\[
\hat{a}_{ij}^c \in [0,1], \quad \forall i \in N; j \in H \quad (e)
\]

Theorem 1 below transforms model $P_1$ to an equivalent linear programming model $P_2$.

**Theorem 1:** Non-linear programming model $P_1$ can be transformed into the following linear programming model $P_2$:

\[
\min \frac{1}{mnh} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} \left[ a_{ij}^k - \hat{a}_{ij}^c \right] \]

subject to

\[
\sum_{k=1}^{m} \hat{a}_{ij}^c \leq c_{ij}, \quad \forall i \in M; j \in H \quad (a)
\]

\[
\sum_{k=1}^{m} \hat{a}_{ij}^c \leq c_{ij}, \quad \forall i \in M; j \in H \quad (b)
\]

\[
\sum_{k=1}^{m} \hat{a}_{ij}^c \geq \alpha \quad (c)
\]

\[
\hat{a}_{ij}^c \leq d_{ij}, \quad \forall i \in M; j \in H \quad (d)
\]

\[
\hat{a}_{ij}^c \in [0,1], \quad \forall i \in M; j \in H \quad (e)
\]

\[
\hat{a}_{ij}^c = \sum_{k=1}^{m} \alpha_k \cdot \hat{a}_{ij}^k , \quad \forall i \in N; j \in H \quad (f)
\]

\[
\hat{a}_{ij}^c = \theta^t \cdot a_{ij}^t + (1-\theta^t) \sum_{i=1}^{m} w_i^t \cdot a_{ij}^t , \quad \forall i \in N; j \in H \quad (g)
\]

\[
\hat{a}_{ij}^c \leq d_{ij}, \quad \forall i \in M; j \in H \quad (h)
\]

\[
\hat{a}_{ij}^c = \sum_{k=1}^{m} \alpha_k \cdot \hat{a}_{ij}^k , \quad \forall i \in N; j \in H \quad (i)
\]

\[
\hat{a}_{ij}^c, \hat{a}_{ij}^d, c_{ij} \in [0,1], \quad \forall i \in M; j \in H \quad (j)
\]

**Proof:** Constraints (a) and (b) of model $P_2$ imply that $|\hat{a}_{ij}^c - \hat{a}_{ij} | \leq c_{ij}$ ($k \in M; i \in N; j \in H$). Meanwhile, the model $P_2$ objective function cannot achieve its optimal value when $|\hat{a}_{ij}^c - \hat{a}_{ij} | < c_{ij}$ ($k \in M; i \in N; j \in H$). Therefore, it is $|\hat{a}_{ij}^c - \hat{a}_{ij} | = c_{ij}$ ($k \in M; i \in N; j \in H$). According to constraints (d) and (e) of model $P_2$, it is $|\hat{a}_{ij}^c - \hat{a}_{ij} | \leq d_{ij}$ ($k \in M; i \in N; j \in H$). Hence,

\[
1 - \frac{1}{mnh} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} \left[ \hat{a}_{ij}^c - \hat{a}_{ij} \right] \geq \alpha \quad \text{holds. In addition, constraints (h) and (i) of model } P_2 \text{ ensure that } |\hat{a}_{ij}^c - \hat{a}_{ij} | \leq d_{ij} \quad (k \in M; i \in N; j \in H). \]

Accordingly, model $P_2$ is equivalently converted into model $P_3$.

In model $P_3$, a larger $\alpha$ value and smaller $u^+$ ($k \in M$) values mean that a smaller feasible region. In some cases, the feasible region of model $P_3$ may be empty, which means that model $P_3$ may not be solvable. In the following, we present STDCMOM to analyze the solution of model $P_3$. Recall that $\{\bar{A}, ..., \bar{A}^n\}$ be the adjusted MAAMs associated with $\{A'^+, ..., A'^n\}$, should satisfy the constraints (b)-(e) in model $P_3$. On the other hand, Assumption 2 is verified when the consensus level regarding MAAMs $\{\bar{A}, ..., \bar{A}^n\}$ is maximized, i.e.,

\[
\max \frac{1}{mnh} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} |\hat{a}_{ij}^c - \hat{a}_{ij} | \quad (18)
\]

Similar to the developing of the STDMACM model, the following STDCMOM non-linear programming model, with decision variables $(\bar{A} = (\hat{a}_{ij}^c)_{mn}, \bar{A}^n = (\hat{a}_{ij}^c)_{mn}, \bar{A} = (\hat{a}_{ij}^c)_{mn})$, which is referred to as model $P_3$, is introduced:

\[
\max \frac{1}{mnh} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} |\hat{a}_{ij}^c - \hat{a}_{ij} | \quad (19)
\]

Theorem 2 converts model $P_3$ to an equivalent linear programming model $P_4$.

**Theorem 2:** Non-linear programming model $P_3$ can be transformed to the following linear programming model $P_4$:

\[
\max \frac{1}{mnh} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} |\hat{a}_{ij}^c - \hat{a}_{ij} | \quad (20)
\]

The proof of Theorem 2 is similar to that of Theorem 1, and therefore it is omitted.

**Theorem 3:** Let $CL'$ be the optimal objective function value of model $P_3$ ($P_4$), if $CL' \geq \alpha$ then the feasible solution of model $P_3$ ($P_4$) exists.

**Proof:** Let MAAMs $\{A'^+, ..., A'^n, A'^+\}$ be the optimal solution to model $P_3$. It is clear that MAAMs $\{A'^+, ..., A'^n, A'^+\}$ satisfy all the constraints of model $P_3$. So, the feasible solution of model $P_3$ exists.

Theorem 3 provides an approach to judge whether the predefined consensus level of the STDMACM model $P_3$ can be achieved.

**B.3. Interactive social trust driven consensus reaching process and its algorithm**

In existing works, the optimization-based consensus model is often used in the feedback adjustment process to generate assessments-modifications suggestions so as to develop an iterative consensus reaching process [21]. Following this basic idea, we propose an interactive social trust driven consensus reaching process, in which the optimal adjusted assessments generated from the optimization-based social trust driven consensus models (STDCMOM or STDMACM) are used as references for guiding the assessments-modifications suggestions. The interactive social trust driven consensus reaching process is presented below.
Firstly, the benchmark MAAMs $\hat{A}^t = (\hat{a}^t)_{ijk}$ ($k \in M$) are obtained using Eq. (9). Taking $[A^t = (a^t)_{ijk}, \hat{A}^t = (\hat{a}^t)_{ijk}, u^t, \alpha, \theta^t, \alpha] \ (k \in M)$ as the input of model $P_z$, then the optimal adjusted MAAMs $[A^{t\prime} = (a^{t\prime})_{ijk}, \ldots, A^{t\prime\prime} = (a^{t\prime\prime})_{ijk}]$ are generated. The following two cases are considered when constructing updated MAAMs $\hat{A}^t = (\hat{a}^t)_{ijk}$ ($k \in M$):

**Case A:** $\text{CL}(A^{t\prime}, \ldots, A^{t\prime\prime}) < \alpha$.

(i) If $a^t_k < a^{t\prime}_k$, then decision maker $e_k$ is advised to increase his/her assessment value of alternative $x_i$ with respect to attribute $b_j$;

(ii) If $a^t_k > a^{t\prime}_k$, then decision maker $e_k$ is advised to decrease the assessment value of alternative $x_i$ with respect to attribute $b_j$;

(iii) If $a^t_k = a^{t\prime}_k$, then decision maker $e_k$ is advised to remain the assessment value of alternative $x_i$ with respect to attribute $b_j$ unchanged.

Summarizing, decision maker $e_k$ is advised:

$$\hat{a}^t_k \in [\min\{a^t_k, a^{t\prime}_k\}, \max\{a^t_k, a^{t\prime}_k\}]$$

**Case B:** $\text{CL}(A^{t\prime}, \ldots, A^{t\prime\prime}) \geq \alpha$.

In this case, taking $[A^t = (a^t)_{ijk}, \hat{A}^t = (\hat{a}^t)_{ijk}, u^t, \alpha, \theta^t, \alpha] \ (k \in M)$ as the input of model $P_z$ obtains the optimal adjusted MAAMs $[A^{t\prime\prime} = (a^{t\prime\prime})_{ijk}, \ldots, A^{t\prime\prime\prime\prime} = (a^{t\prime\prime\prime\prime})_{ijk}]$. Similar to Case A, when constructing updated MAAMs $\hat{A}^t = (\hat{a}^t)_{ijk}$ ($k \in M$), decision maker $e_k$ is advised:

$$\hat{a}^t_k \in [\min\{a^t_k, a^{t\prime\prime\prime\prime}_k\}, \max\{a^t_k, a^{t\prime\prime\prime\prime}_k\}]$$

Based on the above, the below describes the proposed interactive social trust driven consensus reaching process while its corresponding algorithmic representation is given in Table 2.

Based on social trust network analysis, the decision makers’ weights are determined, and the social trust based feedback adjustment mechanism is designed.

Consensus level among decision makers is computed, and if the threshold of consensus is achieved, then the selection process is activated to obtain the preference ordering of alternatives; otherwise, the below social trust driven consensus reaching model with minimum adjustments is processed to support the decision makers in modifying their MAAMs until the threshold of consensus is achieved or a predefined maximum number of consensus rounds is achieved.

STDCMOM is used to generate the maximum consensus level among decision makers. If the obtained maximum consensus level is acceptable, then the STDMACM is used to generate assessments-modifications suggestions; otherwise, the assessments-modifications suggestions generated from the STDCM are used as references for supporting the consensus reaching.

<table>
<thead>
<tr>
<th>Table 2: Consensus algorithm I</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> MAAMs $A^t = (a^t)<em>{ijk}$ ($k \in M$), socimatrix $T = (t^i)</em>{m\times n}$, consensus threshold $\alpha$, self-confidence levels $\theta^t$, $\ldots$, $u^t$ ($k \in M$), weight vector over attributes $\beta$, and maximum consensus round $r_{\text{max}}$</td>
</tr>
<tr>
<td><strong>Output:</strong> The adjusted MAAMs $\hat{A}^t = (\hat{a}^t)<em>{ijk}$ ($k \in M$), the adjusted collective MAAM $\hat{X} = (\hat{x})</em>{ijk}$, and the preference ordering of alternatives $\alpha’ = (a_1’, a_2’, \ldots, a_k’)’$</td>
</tr>
</tbody>
</table>

**Step 1:** Let $r = 0$, and $A^{t\prime} = A^t$. Based on Eq. (7), the decision makers’ weights are obtained: $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, where $\lambda_i = IC_1 \sum_{i=1}^{n} IC_i$ and $IC_i = \frac{1}{m} \sum_{j=1}^{m} t_{ij}$. Meanwhile, using Eq. (8) to obtain the weighting vector $w = (w_1, w_2, \ldots, w_k)$ ($k \in M$).

**Step 2:** According to Eq. (10), the collective MAAM is generated: $A^{t\prime\prime} = (a^{t\prime\prime})_{ijk}$ where $a^{t\prime\prime} = \sum_{i=1}^{m} \lambda_i \cdot a_{ijk}$. Using Eq. (12) the consensus level among decision makers is computed: $\text{CL}(A^{t\prime\prime}, \ldots, A^{t\prime\prime\prime\prime}) \geq \alpha$ or $r > r_{\text{max}}$, go to Step 5; otherwise, go to Step 3.

**Step 3:** Apply STDCMOM $P_z$ to yield optimal MAAMs $[A^{t\prime\prime}, \ldots, A^{t\prime\prime\prime\prime}]$.

**Step 4:** If $\text{CL}(A^{t\prime\prime}, \ldots, A^{t\prime\prime\prime\prime}) < \alpha$, then suggest decision makers to generate updated MAAMs $A^{t\prime\prime\prime\prime\prime} = (a^{t\prime\prime\prime\prime\prime})_{ijk}$ ($k \in M$) with $a^{t\prime\prime\prime\prime\prime} \in [\min\{a^{t\prime\prime\prime\prime\prime}_k, a^{t\prime\prime\prime\prime\prime}_k\}, \max\{a^{t\prime\prime\prime\prime\prime}_k, a^{t\prime\prime\prime\prime\prime}_k\}]$; otherwise apply STDMACM $P_z$ to yield optimal adjusted MAAMs $[A^{t\prime\prime\prime\prime\prime}, \ldots, A^{t\prime\prime\prime\prime\prime\prime\prime}]$ and suggest decision makers to generate updated MAAMs $A^{t\prime\prime\prime\prime\prime\prime\prime\prime} = (a^{t\prime\prime\prime\prime\prime\prime\prime\prime})_{ijk}$ ($k \in M$) with $a^{t\prime\prime\prime\prime\prime\prime\prime\prime} \in [\min\{a^{t\prime\prime\prime\prime\prime\prime\prime\prime}_k, a^{t\prime\prime\prime\prime\prime\prime\prime\prime}_k\}, \max\{a^{t\prime\prime\prime\prime\prime\prime\prime\prime}_k, a^{t\prime\prime\prime\prime\prime\prime\prime\prime}_k\}]$.

Let $r = r + 1$, and go to Step 2.

**Step 5:** Let $X = A^{t\prime}$ and $\hat{X} = A^{t\prime\prime}$. Using Eqs. (2) and (3) transforms $\hat{X}$ into $\hat{V}$. Employ Eq. (4) to obtain $p’ = (p_1, p_2, \ldots, p_k)^T$ from $\hat{V} = (\hat{v})_{ijk}$. Apply Eq. (5) to derive the preference ordering $\alpha’ = (a_1’, a_2’, \ldots, a_k’)’$ from $p’$.

**Step 6:** Output $[A^t, \ldots, \hat{A}^t]$ and $\alpha’$.

IV. ILLUSTRATIVE EXAMPLE

In this section, an illustrative example is provided to show the applicability of the proposed interactive social trust driven consensus reaching process. Five decision makers (denoted as $E = \{e_1, e_2, \ldots, e_5\}$), three alternatives (denoted as $X = \{x_1, x_2, \ldots, x_5\}$) and three attributes (denoted as $B = \{b_1, b_2, b_3\}$) are considered with $b_1$ and $b_2$ being benefit attributes while $b_3$ being a cost attribute. The following parameter values are predefined: $\alpha = 0.93$, $\theta = 0.75$, $\theta^2 = 0.85$, $\theta^3 = 0.8$, $\theta^4 = 0.65$, $\theta^5 = 0.75$, $u^1 = 0.05$, $u^2 = 0.02$, $u^3 = 0.05$, $u^4 = 0.05$, $u^5 = 0.05$, and $r_{\text{max}} = 4$.

The MAAMs $A^t = (a^t)_{ijk}$ offered by the five decision makers are listed in Appendix (See Table A1).

The social graph (denoted as $G$) among the five decision makers $E = \{e_1, e_2, \ldots, e_5\}$ is shown in Fig. 2 and its corresponding sociomatrix being:

$$T = \begin{bmatrix}
0.6 & 0.7 & 0.55 & 0.9 \\
0.7 & 0.4 & 0.85 & 0.3 \\
0.35 & 0.6 & 0.4 & - \\
0.6 & - & 0.9 & - \\
\end{bmatrix}$$

Fig. 2. The graph $G$ associated with the five decision makers.
(1) Social trust network analysis

According to Eq. (6), it is \( IC^1 = 0.325, IC^2 = 0.35, IC^3 = 0.425, IC^4 = 0.525, \) and \( IC^5 = 0.75. \) The weight of decision maker \( e_i \) is obtained using Eq. (7):

\[
\lambda_i^e = \frac{0.325}{0.325 + 0.35 + 0.425 + 0.525 + 0.75} = 0.325 = 0.1368.
\]

Similarly, it is: \( \lambda = (0.1368, 0.1474, 0.1789, 0.2211, 0.3158)^T. \)

According to Eq. (8), it is:

\[
w_i^e = 0,  \\
w_i^e = 0.6 \left\{ 0.6 + 0.7 + 0.55 + 0.9 \right\} = 0.325, \\
w_i^e = 0.7 \left\{ 0.6 + 0.7 + 0.55 + 0.9 \right\} = 0.285, \\
w_i^e = 0.55 \left\{ 0.6 + 0.7 + 0.55 + 0.9 \right\} = 0.2, \) and
\[
w_i^e = 0.3257, \\
w_i^e = 0.6 \left\{ 0.6 + 0.7 + 0.55 + 0.9 \right\} = 0.3273. 
\]

Thus, \( w^e = (0.2182, 0.2545, 0.2, 0.3273)^T. \) Similarly, it is:

\[
w^e = (0.359, 0.2051, 0.4359)^T, \\
w^e = (0, 0.2308, 0, 0.3333, 0.4359)^T, \\
w^e = (0, 0.2593, 0.4444, 0, 0.2963)^T, \) and \( w^e = (0.4, 0, 0.6, 0)^T. \)

(2) Application of social trust driven consensus model with minimum adjustments

Based on Eq. (10), the collective MAAM is:

\[
\text{Table 3: MAAM } A_e
\]

\[
\begin{array}{lcl}
b_1 & b_2 & b_3 \\
x_1 & 0.3558 & 0.6484 & 0.3863 \\
x_2 & 0.52 & 0.3837 & 0.2763 \\
x_3 & 0.5984 & 0.3311 & 0.5311 \\
\end{array}
\]

From Eq. (12), the consensus level among decision makers is: \( CL(A^1, \ldots, A^5) = 0.973, \) which is not acceptable because \( CL(A^1, \ldots, A^5) = 0.973 < \alpha = 0.93. \) Based Eq. (9), the reference MAAMs \( A^i (i = 1, 2, \ldots, 5) \) are obtained, which are provided in Appendix (See Table A2).

Using STDCMOM (model \( P_1 \)), the optimal adjusted MAAMs \( (A^1, \ldots, A^5) \) are computed (Table A3, Appendix). Because \( CL(A^1, \ldots, A^5) = 0.8937 < \alpha = 0.93, \) decision makers are advised: your update MAAMs should be \( A^i = (\alpha^i)_{1:5} (i = 1, \ldots, 5) \) with elements verifying \( a_{ij}^i = \min(a_{ij}^e, a_{ij}^3), \max(a_{ij}^e, a_{ij}^4) \) \( (i = 1, \ldots, 5; \) \( i, j = 1, 2, 3) \). Because \( A^i = (\alpha^i)_{1:5} (i = 1, \ldots, 5) \) satisfy Assumption 1, decision makers \( e_i \) will accept \( A^i = (\alpha^i)_{1:5} \) and the updated MAAMs will be \( A^i = A^i (i = 1, 2, \ldots, 5) \).

The above process is repeated again. Applying STDCMOM (model \( P_2 \)), it is \( CL(A^1, \ldots, A^5) = 0.9520 > \alpha = 0.93. \) Thus, using STDMACM (model \( P_2 \)) the optimal adjusted MAAMs \( (A^1, \ldots, A^5) \) (Table A4, Appendix) are generated and provided as references for decision makers \( e_i \) to construct new MAAMs \( A^2 : a_{ij}^2 = \min(a_{ij}^e, a_{ij}^3), \max(a_{ij}^e, a_{ij}^4) \) \( (i = 1, \ldots, 5; \) \( i, j = 1, 2, 3) \). Because \( A^2 (i = 1, 2, \ldots, 5) \) satisfy Assumption 1, decision makers \( e_i \) provide MAAMs: \( A^2 = A^2 (i = 1, 2, \ldots, 5). \) Moreover, we have that \( CL(A^2, \ldots, A^2) = 0.93 \) according to Eq. (12), which indicates that Assumption 2 is also verified, i.e. the threshold consensus level among the five decision makers is achieved.

(3) Selection process

Firstly, the collective MAAM \( A^{c2} \) (Table 4) is obtained from MAAMs \( \{A^1, \ldots, A^5\} \) via Eq. (10).

\[
\text{Table 4: MAAM } A^{c2}
\]

\[
\begin{array}{lcl}
b_1 & b_2 & b_3 \\
x_1 & 0.3669 & 0.6792 & 0.3791 \\
x_2 & 0.4477 & 0.4154 & 0.3026 \\
x_3 & 0.6366 & 0.3467 & 0.4712 \\
\end{array}
\]

Then, according to Eqs. (2) and (3), MAAM \( A^{c2} \) is converted into MAAM \( V^{c2}: \)

\[
\text{Table 5: MAAM } V^{c2}
\]

\[
\begin{array}{lcl}
b_1 & b_2 & b_3 \\
x_1 & 0 & 1 & 0.5463 \\
x_2 & 0.2996 & 0.2066 & 1 \\
x_3 & 1 & 0 & 0 \\
\end{array}
\]

According to Eq. (4), it is:

\[
\begin{array}{lcl}
sv' = 1 & 1 & 1 \\
\end{array}
\]

So, the alternatives ordering will be \( s' = (1, 2, 3)^T \) and \( x_1 > x_2 > x_3. \)

V. SIMULATION AND COMPARISON ANALYSIS

This section analyzes the performance of the proposed STDCM with minimum adjustments through simulation and comparison experiments. The assessment of the efficiency of consensus models is often based on the following two criteria [37]: (i) the number of consensus rounds, \( r \); and (ii) the consensus success ratio, \( s \).

Simulation method I, given in Table 6, is designed to study the efficiency of the STDCM with respect to criteria \( r \) and \( s \). Both the MAAMs and social trust networks are randomly generated, and the values for \( r \) and \( s \) associated are then generated using Consensus algorithm I with decision makers implementing the assessments-modifications suggestions when they satisfy Assumption 1.

\[
\text{Table 6: Simulation method I}
\]

\[
\begin{array}{lcl}
\text{Input: } m, n, h, G, \alpha, \theta \epsilon (k \in M), \mu \epsilon (k \in M), r_{max}. \\
\text{Output: } r \text{ and } s. \\
\end{array}
\]

Step 1: Generate a set of MAAMs \( A^i = (a^i)_{1:5} (k \in M) \), where \( a^i \) is randomly generated using the uniform distribution over the interval \([0, 1]. \)
Sociomatrix \( T = (t^i)_{1:5} \) is generated as follows: if there exists trust from \( e_i \) to \( e_j \) in \( G, \) then \( t^i_j \) is randomly generated using the uniform distribution over the interval \([0, 1]; \) otherwise \( t^i_j = 0. \)

Step 2: Take \( A^i = (a^i)_{1:5}, \alpha, \theta, \mu, \) and \( r_{max} \) as inputs of the Consensus algorithm I to yield \( r \) and \( \theta^i = (a^i)_{1:5} (k \in M). \) If \( CL(A^1, \ldots, A^5) \geq \alpha, \) then \( s = 1; \) otherwise, \( s = 0. \)

Step 3: Output \( r \) and \( s. \)

In existing social network consensus model (SNCM), the impact of social trust relationships on the assessments-modifications
suggestions is not considered (e.g., [48, 49]). Removing constraints (c) and (d) from model \( P \) leads to an adjustment-minimum consensus model without considering the impact of social network structure on the assessments-modifications suggestions, which is referred to as model \( P' \). The SNCM consensus algorithm is provided in Table 7, and it is named Consensus algorithm II herein.

Table 7: Consensus algorithm II

| Input: \( A^i(\alpha^i)_{m \times n}, T=(t_{ij})_{m \times n}, \alpha, \theta^i, u^i \), and \( r_{max} \). |
| Output: The adjusted MAAMs \( \bar{A}^i=(\bar{a}^i)_{m \times n} \) (\( k \in M \)) and \( \bar{A}^j=(\bar{a}^j)_{m \times n} \), the preference ordering of alternatives \( o'=(o'_1, \ldots, o'_n) \). |
| Step 1: Same as Step 1 in Consensus algorithm I. |
| Step 2: According to Eq. (10), the collective MAAM is generated: \( A'^r=(\bar{a}^r)^{m \times n} \) where \( a'^r_{ij}=\sum_{k=1}^{n} \lambda^r_k \cdot a^r_{ij} \). Apply Eq. (12) to calculate the consensus level among decision makers \( CL(A^r, \ldots, A'^r) \). If \( CL(A^r, \ldots, A'^r) \geq \alpha \), go to Step 5; otherwise, go to Step 3. |
| Step 3: Apply model \( P \) to obtain the optimal MAAMs \( \{A'^r, \ldots, A'^r\} \). |
| Step 4: Based on Eq. (9), it is: \( a^r_{ij}=\theta^r \cdot a^r_{ij}+(1-\theta^r) \sum_{k=1}^{n} \mu^r_k \cdot a^r_{ij} \) (\( k \in M; i \in N; j \in H \)). Update MAAMs \( A'^r(\alpha^r)_{m \times n} \) (\( k \in M \)) are constructed as follows:

\[
\begin{align*}
\hat{a}^r_{ij} &= a^r_{ij}, \quad \text{if } |a^r_{ij}-a^r_{ij}| \leq u^r \\
\hat{a}^r_{ij} &= a^r_{ij}, \quad \text{otherwise}
\end{align*}
\]

Let \( r=r+1 \), and go to Step 2. |
| Step 5: Let \( \bar{A}^i=A'^r \) (\( k \in M \)) and \( \bar{A}^j=A'^r \). Apply Eqs. (2) and (3) to generate normalized MAAM \( V'^r \) from \( A'^r \). Employ Eq. (4) to obtain \( p'^r=(p'^r_1, p'^r_2, \ldots, p'^r_n) \) from \( A'^r \). Use Eq. (5) to generate the preference ordering \( o'=(o'_1, o'_2, \ldots, o'_n) \) from \( p'^r \). |
| Step 6: Output \( \{\bar{A}^i, \bar{A}^j, \bar{A}^k\} \) and \( o' \). |

Simulation method II is designed to obtain results from existing SNCM lacking the impact of social trust relationships on the assessments-modifications with respect to criteria \( r \) and \( s \). Simulation method II is designed by replacing Consensus algorithm I with Consensus algorithm II in Step 2 of Simulation method I. Specifically, Step 2 of Simulation method II reads:

"Take \( A^k(\alpha^k)_{m \times n} (k=1,2,\ldots,m), T=(t_{ij})_{m \times n}, \alpha, \theta^k, \mu^k \), and \( r_{max} \) as inputs of Consensus algorithm II to yield \( r \) and \( \bar{A}^k=(\bar{a}^k)_{m \times n} \) (\( k=1,2,\ldots,m \)). If \( CL(\bar{A}^1, \ldots, \bar{A}^n) \geq \alpha \), then \( s=1 \); otherwise \( s=0 \)."

In the simulation and comparison analysis, we set \( u^r=u \) and \( \theta^r=\theta \) \( \forall k \in M \), with \( G \) being the graph used in the illustrate example section.

- **Simulation method I data sets:**
  - (S1) \( m=5 \), \( G \) (see Fig. 2), \( n=3 \), \( h=3 \), \( \alpha=0.9 \), \( u=\{0.005, 0.01, 0.015, 0.02, 0.025, \ldots, 0.095, 0.1\} \), and \( \theta=\{0.15, 0.2, 0.25, 0.3, 0.35, \ldots, 0.85, 0.9\} \);
  - (S2) \( m=5 \), \( G \) (see Fig. 2), \( n=3 \), \( h=3 \), \( \alpha=0.92 \), \( u=\{0.005, 0.01, 0.015, 0.02, 0.025, \ldots, 0.095, 0.1\} \), and \( \theta=\{0.15, 0.2, 0.25, 0.3, 0.35, \ldots, 0.85, 0.9\} \).

Simulation method I is run 1000 times to obtain average values of \( r \) and \( s \) under different parameter combinations. Results for data sets (S1) and (S2) are depicted in Figs. 3 and 4, respectively.

- **Simulation method II data sets:**
  - (C1) \( m=5 \), \( G \) (see Fig. 2), \( n=3 \), \( h=3 \), \( \theta=0.65 \), \( u=\{0.08, 0.11, 0.14, 0.17, 0.2, 0.23, 0.26\} \), and \( \alpha=\{0.85, 0.87, 0.89, 0.91, 0.92, 0.94\} \);
  - (C2) \( m=5 \), \( G \) (see Fig. 2), \( n=3 \), \( h=3 \), \( \theta=0.8 \), \( u=\{0.08, 0.11, 0.14, 0.17, 0.2, 0.23, 0.26\} \), and \( \alpha=\{0.85, 0.87, 0.89, 0.91, 0.92, 0.94\} \).

Simulation method II is run 1000 times to produce average values of \( r \) and \( s \), with results for data sets (C1) and (C2) depicted in Figs. 5 and 6, respectively.
often requires 1-3 consensus rounds to achieve the pre-determined consensus level and its consensus success ratio is 1 for most of the parameter combinations.

(2) The STDCM average value \( r \) decreases when the \( u \) value increases or the \( \theta \) value decreases. This implies that the speed to achieve the pre-determined consensus level will accelerate when decision makers have large tolerance degrees or small self-confidence levels.

(3) On average, the consensus rounds in the STDCM are lower than in the SNCM. Moreover, the STDCM consensus success ratio is higher than the SNCM consensus success ratio. These results imply that by considering the impact of social trust network on assessments-modifications suggestions, the STDCM enhances both the convergence speed and the success ratio to achieve the pre-determined consensus.

VI. CONCLUSION

This study develops a STDCM with minimum adjustments in SNGDM considering the impact of the social trust relationships on the assessments-modifications suggestions. The main research results within this study are:

(1) The design of a social trust based feedback adjustment mechanism that is based on the assumption that decision makers will accept the assessments-modifications suggestions when they are close to their trusted decision makers’ assessments in a social trust network.

(2) The development of two programming models, STDCMOM and STDMACM, to generate assessments-modifications suggestions to support and enhance the consensus reaching in the SNGDM.

(3) The algorithmic construction of an interactive consensus reaching process based on STDMACM and STDMCM, which has been validated with a simulation and comparison analysis with existing consensus reaching models in terms of consensus convergence rate and consensus success ratio.

Meanwhile, the following three research directions may be of interest to researchers on this area:

(1) In practice, decision makers may prefer to express their assessments using a linguistic way rather than a numerical way [50]. It may be interesting to extend the findings of this study to linguistic decision context.

(2) How to set the parameters is an open problem in the consensus reaching model. We argue that the use of data-driven preference learning method may be a promising way to estimate the parameters in consensus reaching model.

(3) Decision makers’ behaviors and psychological foundations have a significant impact on the efficiency of consensus-based decision-making. Thus, to examine game and non-cooperative manipulation behaviors [25, 51-53], and psychological foundations (such as prospect theory) in the STDCM is a promising avenue to investigate.

Appendix

Table A1: MAAMs \( A^1 - A^5 \)

<table>
<thead>
<tr>
<th>( A^1 )</th>
<th>( A^2 )</th>
<th>( A^3 )</th>
<th>( A^4 )</th>
<th>( A^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.35 0.4 0.55</td>
<td>0.25 0.9 0.15</td>
<td>0.05 0.6 0.3</td>
<td>0.9 0.6 0.35</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.2 0.55 0.25</td>
<td>0.15 0.8 0.35</td>
<td>0.3 0.5 0.5</td>
<td>0.6 0.1 0.1</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.3 0.05 0.25</td>
<td>0.95 0.75 0.2</td>
<td>0.7 0.4 0.75</td>
<td>0.75 0.5 0.15</td>
</tr>
</tbody>
</table>

Table A2: MAAMs \( \tilde{A}^1 - \tilde{A}^5 \)

<table>
<thead>
<tr>
<th>( \tilde{A}^1 )</th>
<th>( \tilde{A}^2 )</th>
<th>( \tilde{A}^3 )</th>
<th>( \tilde{A}^4 )</th>
<th>( \tilde{A}^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.3407 0.4745 0.4982</td>
<td>0.2460 0.8508 0.1990</td>
<td>0.1290 0.6226 0.3138</td>
<td>0.6362 0.6376 0.3396</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.2809 0.5134 0.2639</td>
<td>0.2063 0.7413 0.3427</td>
<td>0.3504 0.4654 0.4446</td>
<td>0.5436 0.2413 0.2005</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.3916 0.1370 0.3314</td>
<td>0.8713 0.6590 0.2687</td>
<td>0.6887 0.3967 0.7021</td>
<td>0.7241 0.4657 0.3308</td>
</tr>
</tbody>
</table>

Table A3: MAAMs \( A^{iv} - A^{iv} \)

<table>
<thead>
<tr>
<th>( A^{iv} )</th>
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<th>( A^{iv} )</th>
<th>( A^{iv} )</th>
<th>( A^{iv} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.3669 0.5245 0.4482</td>
<td>0.266 0.8308 0.2190</td>
<td>0.179 0.6726 0.3638</td>
<td>0.5862 0.6792 0.3791</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.3309 0.4634 0.3026</td>
<td>0.2263 0.7213 0.3227</td>
<td>0.4154 0.4154 0.3946</td>
<td>0.4944 0.2913 0.2505</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.4416 0.187 0.3814</td>
<td>0.8513 0.639 0.2887</td>
<td>0.6387 0.3467 0.6521</td>
<td>0.6741 0.4156 0.3808</td>
</tr>
</tbody>
</table>

Table A4: MAAMs \( A^{iv} - A^{iv} \)

<table>
<thead>
<tr>
<th>( A^{iv} )</th>
<th>( A^{iv} )</th>
<th>( A^{iv} )</th>
<th>( A^{iv} )</th>
<th>( A^{iv} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.3669 0.5665 0.4248</td>
<td>0.2874 0.7885 0.2606</td>
<td>0.2637 0.6754 0.3706</td>
<td>0.5034 0.6792 0.3791</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.3309 0.4428 0.3026</td>
<td>0.251 0.6577 0.3154</td>
<td>0.4154 0.4154 0.3562</td>
<td>0.4477 0.3474 0.2734</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.5423 0.2562 0.3814</td>
<td>0.8192 0.5654 0.3124</td>
<td>0.6387 0.3467 0.5723</td>
<td>0.6741 0.3875 0.3988</td>
</tr>
</tbody>
</table>
REFERENCES


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