A learning procedure to estimate missing values in fuzzy preference relations based on additive consistency

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Abstract. In decision-making, information is usually provided by means of fuzzy preference relations. However, there may be cases in which experts do not have an in-depth knowledge of the problem to be solved, and thus their fuzzy preference relations may be incomplete, i.e. some values may not be given or may be missing. In this paper we present a procedure to find out the missing values of an incomplete fuzzy preference relation using the values known. We also define an expert consistency measure, based on additive consistency property. We show that our procedure to find out the missing values maintains the consistency of the original, incomplete fuzzy preference relation provided by the expert. Finally, to illustrate all this, an example of the procedure is presented.

Keywords: *Decision-making, fuzzy preference relations, missing values, consistency, additive consistency, incomplete information*

1 Introduction

Decision-making procedures are increasingly being used in various different fields for evaluation, selection and prioritisation purposes, that is, making preference decisions about a set of different choices. Furthermore, it is also obvious that the comparison of different alternative actions according to their desirability in decision problems, in many cases, cannot be done using a single criterion or one person. Indeed, in the majority of decision making problems, procedures have been established to combine opinions about alternatives related to different points of view. These procedures are based on pair comparisons, in the sense that processes are linked to some degree of credibility of preference of one alternative over another. Many different representation formats can be used to express preferences. *Fuzzy preference relation* is one of these formats, and it is usually used by an expert to provide his/her preference degrees when comparing pairs of alternatives [1, 3, 5, 7].

Since each expert is characterised by their own personal background and experience of the problem to be solved, experts' opinions may differ substantially (there are plenty of educational and cultural factors that influence an expert's preferences). This diversity of experts could lead to situations where some of them would not be able to efficiently express any kind of preference degree between two or more of the available options. Indeed, this may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others. In these situations such an expert is forced to provide an *incomplete fuzzy preference relation* [9].

Usual procedures for multi-person decision-making problems correct this lack of knowledge of a particular expert using the information provided by the rest of the experts together with aggregation procedures [6]. These approaches have several disadvantages. Among them we can cite the requirement of multiple experts in order to learn the missing value of a particular one. Another drawback is that these procedures normally do not take into account the differences between experts' preferences, which could lead to the estimation of a missing value that would not naturally be compatible with the rest of the preference values given by that expert. Finally, some of these missing information-retrieval procedures are interactive, that is, they need experts to collaborate in "real time", an option which is not always possible.

Our proposal is quite different to the above procedures. We put forward a procedure which attempts to find out the missing information in an expert's incomplete fuzzy preference relation, using only the preference values provided by that particular expert. By doing this, we assure that the reconstruction of the incomplete fuzzy preference relation is compatible with the rest of the information provided by that expert. In fact, the procedure we propose in this paper is guided by the expert's consistency which is measured taking into account only the provided preference values. Thus, an important objective in the design of our procedure is to maintain experts' consistency levels. In particular, in this paper we use the additive consistency property [4] to define a consistency measure of the expert's information.

In order to do this, the paper is set out as follows. Section 2 presents some preliminaries on the additive consistency property. In Section 3, a new consistency measure and the learning procedure are described. We also include a brief discussion of the possible situations in which the procedure will be successful in discovering all the missing values and we provide the sufficient conditions that will guarantee this. In Section 4, we present a simple but illustrative example of how the iterative procedure to discover the missing values in incomplete fuzzy preference relations works. Finally, our concluding remarks and topics for possible future research are pointed out in Section 5.

2 Preliminaries: Additive Consistency

Preference relations are one of the most common representation formats of information used in decision-making problems because they are a useful tool in modelling decision processes, above all when we want to aggregate experts' preferences into group preferences [3–5, 8]. In particular, *fuzzy preference relations* have been used in the development of many important decision-making procedures.

Definition 1 *[5, 7] A fuzzy preference relation P on a set of alternatives X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function*

$$\mu_P : X \times X \longrightarrow [0, 1]$$

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ij})$ being $p_{ij} = \mu_P(x_i, x_j) \;\; \forall i, j \in \{1, \ldots, n\}$ interpreted as the preference degree or intensity of the alternative x_i over x_j: $p_{ij} = 1/2$ indicates indifference between x_i and x_j ($x_i \sim x_j$), $p_{ij} = 1$ indicates that x_i is absolutely preferred to x_j, and $p_{ij} > 1/2$ indicates that x_i is preferred to x_j ($x_i \succ x_j$). Based on this interpretation we have $p_{ii} = 1/2 \;\; \forall i \in \{1, \ldots, n\}$ ($x_i \sim x_i$).

The previous definition does not imply any kind of consistency. In fact, preferences expressed in the fuzzy preference relation can be contradictory. As studied in [4], to make a rational choice, a set of properties to be satisfied by such fuzzy preference relations have been suggested. Transitivity is one of the most important properties concerning preferences, and it represents the idea that the preference value obtained by directly comparing two alternatives should be equal to or greater than the preference value between those two alternatives obtained using an indirect chain of alternatives [2]. One of these properties is the *additive transitivity* [8]:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \;\; \forall i, j, k \in \{1, \ldots, n\} \tag{1}$$

or equivalently:

$$p_{ij} + p_{jk} - 0.5 = p_{ik} \;\; \forall i, j, k \in \{1, \ldots, n\} \tag{2}$$

In this paper, we will consider a fuzzy preference relation to be "additive consistent" when for every three options in the problem $x_i, x_j, x_k \in X$ their associated preference degrees p_{ij}, p_{jk}, p_{ik} fulfil Equation 2. An additive consistent fuzzy preference relation will be referred to as consistent throughout this paper, as this is the only transitivity property we are considering.

3 A learning procedure to estimate missing values in fuzzy preference relations based on additive consistency

As we have already mentioned, missing information is a problem that we have to deal with because usual decision-making procedures assume that experts are able to provide preference degrees between any pair of possible alternatives. We note that a missing value in a fuzzy preference relation is not equivalent to a lack of preference of one alternative over another. In fact, a missing value may be the result of the incapacity of an expert to quantify the degree of preference of one alternative over another, and thus the expert decides not to give a preference value to maintain the consistency of the values provided. In such cases, these missing values can be estimated from the existing information using, as a guidance criterion, the consistency degree of that information.

To do this, in this section, we firstly give a definition of a consistency measure of a fuzzy preference relation based on the additive consistency property. We will, then, design the learning procedure to estimate missing values from existing ones. Finally, we will provide sufficient conditions that guarantee the success of the learning procedure in estimating all the missing values of an incomplete fuzzy preference relation.

3.1 Consistency Measure

Equation 2 can be used to calculate the value of a preference degree p_{ik} using other preference degrees in a fuzzy preference relation. In fact,

$$cp_{ik}^{j} = p_{ij} + p_{jk} - 0.5 \tag{3}$$

where cp_{ik}^{j} means the calculated value of p_{ik} via j, that is, using p_{ij} and p_{jk}. Obviously, when the information provided in a fuzzy preference relation is completely consistent then cp_{ik}^{j}, $\forall j \in \{1, \ldots, n\}$ and p_{ik} coincide. However, the information given by an expert does not usually fulfil Equation 2. In such cases, the value

$$\varepsilon p_{ik} = \frac{\sum_{\substack{j=1 \\ j \neq i,k}}^{n} \left| cp_{ik}^{j} - p_{ik} \right|}{n-2} \tag{4}$$

can be used to measure the error expressed in a preference degree between two options. This error can be interpreted as the consistency level between the preference degree p_{ik} and the rest of the preference values of the fuzzy preference relation. Clearly, when $\varepsilon p_{ik} = 0$ then there is no inconsistency at all, and the higher the value of εp_{ik} the more inconsistent p_{ik} is with respect to the rest of the information.

The *consistency level* for the whole fuzzy preference relation P is defined as follows:

$$CL_P = \frac{\sum_{\substack{i,k=1 \\ i \neq k}}^{n} \varepsilon p_{ik}}{n^2 - n} \tag{5}$$

When $CL_P = 0$ the preference relation P is fully (additive) consistent, otherwise, the higher CL_P the more inconsistent P is.

3.2 A proposal for learning missing values

In the following definitions we express the concept of an incomplete fuzzy preference relation:

Definition 2 *A function $f : X \longrightarrow Y$ is* partial *when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a* total *function.*

Definition 3 *An* incomplete fuzzy preference relation *P on a set of alternatives X is a fuzzy set on the product set $X \times X$ characterized by a* partial *membership function.*

As per this definition, we call a fuzzy preference relation complete when its membership function is a total one. Clearly, the usual definition of a fuzzy preference relation (Section 2) includes both definitions of complete and incomplete fuzzy preference relations. However, as there is no risk of confusion between a complete and an incomplete fuzzy preference relation, in this paper we refer to the first type as simply fuzzy preference relations.

In the case of an incomplete fuzzy preference relations there exists at least a pair of alternatives (x_i, x_j) for which p_{ij} is not known. We will introduce and use throughout this paper the letter x to represent these unknown preference values, i.e. $p_{ij} = x$. We also introduce the following sets:

$$A = \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\} \tag{6}$$

$$MV = \{(i,j) \mid p_{ij} = x, (i,j) \in A\} \tag{7}$$

$$EV = A \setminus MV \tag{8}$$

MV is the set of pairs of alternatives for which the preference degree of the first alternative over the second one is unknown or missing; EV is the set of pairs of alternatives for which the expert provides preference values. Note that we do not take into account the preference value of one alternative over itself, as this is always assumed to be equal to 0.5.

In the case of working with an incomplete fuzzy preference relation, we note that Equation 4 cannot be used. An obvious consequence of this is the need to extend the above definition of CL_P to include cases when the fuzzy preference relation is incomplete. We do this as follows:

$$H_{ik} = \{j \mid (i,j),(j,k) \in EV\} \, \forall i \neq k \tag{9}$$

$$\varepsilon p_{ik} = \frac{\sum_{j \in H_{ik}} \left| cp_{ik}^{j} - p_{ik} \right|}{\# H_{ik}} \tag{10}$$

$$CE_P = \{(\, i,k) \in EV \mid \exists j : (i,j), (j,k) \in EV\} \tag{11}$$

$$CL_P = \frac{\sum_{(i,k) \in CE_P} \varepsilon p_{ik}}{\# CE_P} \tag{12}$$

We call CE_P the *computable error* set because it contains all the elements for which we can compute every εp_{ik}. Clearly, this redefinition of CL_P is an extension of Equation 5. Indeed, when a fuzzy preference relation is complete, both CE_P and A coincide and thus $\# CE_P = n^2 - n$.

To develop the iterative procedure to learn missing values, two different tasks have to be carried out:

A) To establish the elements that can be discovered in each step of the procedure, and
B) To produce the particular expression that will be used to find out a particular missing value.

A) Elements to be learnt in step h

The subset of the missing values MV that can be learnt in step h of our procedure is denoted by LMV_h (*learnable missing values*) and defined as follows:

$$LMV_h = \left\{ (i,k) \in MV \setminus \bigcup_{l=0}^{h-1} LMV_l \;\middle|\; \exists j : (i,j), (j,k) \in EV \cup \left(\bigcup_{l=0}^{h-1} LMV_l \right) \right\} \tag{13}$$

with $LMV_0 = \emptyset$.

When $LMV_{maxIter} = \emptyset$ with $maxIter > 0$ the procedure will stop as there will be no more missing values to learn. Furthermore, if $\bigcup_{l=0}^{maxIter} LMV_l = MV$ then all missing values are learnt and consequently the procedure was successful in the completion of the fuzzy preference relation.

B) Expression to learn the value p_{ik}

In order to learn a particular value p_{ik} with $(i,k) \in LMV_h$, in iteration h, we propose the application of the following three step function:

function learn_p(i,k)

1. $\mathsf{I_{ik}} = \left\{ \mathsf{j} \;\middle|\; (\mathsf{i},\mathsf{j}),(\mathsf{j},\mathsf{k}) \in \mathsf{EV} \cup \left(\bigcup_{\mathsf{l}=0}^{\mathsf{h}-1} \mathsf{LMV_l} \right) \right\}$

2. Calculate $\mathsf{cp}'_{\mathsf{ik}} = \dfrac{\sum_{\mathsf{j} \in \mathsf{I_{ik}}} \mathsf{cp}^{\mathsf{j}}_{\mathsf{ik}}}{\#\mathsf{I_{ik}}}$

3. Make $\mathsf{p_{ik}} = \mathsf{cp}'_{\mathsf{ik}} + \mathsf{z}$ with $\mathsf{z} \in [-\mathsf{CL_P}, \mathsf{CL_P}]$ randomly selected, subject to $0 \leq \mathsf{p_{ik}} + \mathsf{z} \leq 1$

end function

With this procedure, a missing value p_{ik} is estimated using Equation 3 when there is at least one chained pair of known preference values p_{ij}, p_{jk} that allow this. If there is more than one pair of preference values that allow the estimation of p_{ik} using Equation 3 then we use their average value as an estimate of the missing value, cp'_{ik}. Finally, we add a random value $z \in [-CL_P, CL_P]$ to this estimate in order to maintain the consistency level of the expert, but obviously forcing the estimated value to be in the range of the fuzzy preference values $[0, 1]$.

The *iterative learning procedure pseudo-code* is as follows:

```
LMV_0 = ∅
h = 1
while LMV_h ≠ ∅{
for every (i, k) ∈ LMV_h{
        learn_p(i,k)
    }
    h++
}
```

We consider this procedure to be successful when all missing values have been estimated. However, as we have previously mentioned, there are cases when not every missing value of an incomplete fuzzy preference relation can be learnt. In the following, we provide an example illustrating this situation.

3.3 Some missing values cannot be learnt by the iterative procedure

In this section we provide sufficient conditions to assure the learning of all missing values in the incomplete fuzzy preference relation; an example where not all missing values can be learned; and a brief discussion on the role of the additive reciprocity property in the learning process of missing values.

A) Sufficient conditions for learning all missing values

As we will see later, there are cases where all missing information cannot be estimated using our learning procedure. However, to obtain conditions that guarantee that all the missing information in an incomplete fuzzy preference relation could be estimated is of great importance. In the following, we provide sufficient conditions that guarantee the success of the above learning procedure.

It is clear that if a value j exists so that for all $i \in \{1, 2, \ldots, n\}$ both (i, j) and (j, k) do not belong to MV, then all the missing information can be learnt in the first iteration of our procedure ($LMV_1 = MV$) because for every $p_{ik} \in MV$ we can use at least the pair of preference values p_{ij} and p_{jk} to estimate it.

In [4], a different sufficient condition that guarantees the learning of all missing values was given. This condition states that any incomplete fuzzy preference relation can be converted into a complete one when the set of $n - 1$ values $\{p_{12}, p_{23}, \ldots, p_{n-1n}\}$ is known. Another condition, more general than the previous one, is when a set of $n-1$ non-leading diagonal preference values, where each one of the alternatives is compared at least once, is known. This general case includes that one when a complete row or column of preference values is known. However, in these cases the additive reciprocity property is also assumed.

B) Impossibility of learning all the missing values

The following is an illustrative example of an incomplete fuzzy preference relation where our procedure is unable to learn all the missing values.

Suppose an expert provides the following incomplete fuzzy preference relation

$$P = \begin{pmatrix} - & e & e & x & x \\ e & - & x & e & x \\ x & x & - & x & x \\ e & x & x & - & e \\ x & x & e & e & - \end{pmatrix}$$

over a set of five different alternatives, $X = \{x_1, x_2, x_3, x_4, x_5\}$, where x means “a missing value” and e means “a value is known”.

Remark 1. We note that the actual values of the known preference values are not relevant for the purpose of this example.

At the beginning of our iterative procedure we obtain:

$$LMV_1 = \{(1, 4), (2, 3), (2, 5), (4, 2), (4, 3), (5, 1)\}$$

as we can find pairs of preference values that allow us to calculate the missing preference values in these positions. Indeed, the following table shows all the pairs of alternatives that are available to calculate each one of the above missing values:

Missing value (i,k)	Pairs of values to be learnt p_{ik}
$(1,4)$	$(1,2),(2,4)$
$(2,3)$	$(2,1),(1,3)$
$(2,5)$	$(2,4),(4,5)$
$(4,2)$	$(4,1),(1,2)$
$(4,3)$	$(4,1),(1,3);(4,5),(5,3)$
$(5,1)$	$(5,4),(4,1)$

The other missing values cannot be learnt in this first iteration of the procedure. If we substitute all the $x's$ values learnt in this iteration by the number 1 (indicating the step in which they have been learnt) we obtain:

$$P = \begin{pmatrix} - & e & e & 1 & x \\ e & - & 1 & e & 1 \\ x & x & - & x & x \\ e & 1 & 1 & - & e \\ 1 & x & e & e & - \end{pmatrix}$$

In the next iteration, in order to construct the set LMV_2 we can use the values expressed directly by the expert as well as the values learnt in iteration 1. In our case we have $LMV_2 = \{(1,5),(5,2)\}$:

Missing value (i,k)	Pairs of values to be learnt p_{ik}
$(1,5)$	$(1,2),(2,5);(1,4),(4,5)$
$(5,2)$	$(5,1),(1,2);(5,4),(4,2)$

and the incomplete fuzzy preference relation at this point is:

$$P = \begin{pmatrix} - & e & e & 1 & 2 \\ e & - & 1 & e & 1 \\ x & x & - & x & x \\ e & 1 & 1 & - & e \\ 1 & 2 & e & e & - \end{pmatrix}$$

In the next iteration $LMV_3 = \emptyset$. The procedure ends and it does not succeed in the completion of the fuzzy preference relation. The reason for this failure is that the expert did not provide any preference degree of the alternative x_3 over the rest of the alternatives. Fortunately, this kind of situation is not very common in real-life problems, and therefore the procedure will usually be successful in finding out all the missing values. Clearly, if additive reciprocity is also assumed (this is a direct consequence of the additive transitivity property) then the chances of succeeding in estimating all the missing values would increase, as we show next.

C) Additive reciprocity property

In most studies, preference relations are usually assumed to be reciprocal. In particular, *additive reciprocity* is used in many decision models as one of the properties that fuzzy preference relations have to verify [1, 5]. Additive reciprocity is defined as:

$$p_{ij} + p_{ji} = 1 \;\; \forall i, j \in \{1, 2, \ldots, n\} \tag{14}$$

Our iterative procedure does not imply any kind of reciprocity. In fact, it permits missing values in fuzzy preference relations to be estimated when this condition is not satisfied (as we show in Section 4). Furthermore, the procedure itself does not assure that the learnt values will fulfil the reciprocity property.

However, if we assume that the fuzzy preference relation has to be reciprocal, then this would allow some of the missing values that were not possible without it to be estimated. In the previous example all p_{3k} values that it was not possible to estimate could have been easily learnt assuming the additive reciprocity property.

In what follows, we describe how to implement the use of the additive reciprocity in our procedure, and the changes we need to implement to assure that estimated values fulfil this property.

Firstly, we need to guarantee that the incomplete fuzzy preference relation given by the expert fulfils the reciprocity property, i.e. $p_{ij}+p_{ji} = 1 \;\; \forall (i, j), (j, i) \in EV$. This means that the first step of our procedure has to be the computation of those missing values with a known reciprocal one, i.e.

$$p_{ij} \leftarrow 1 - p_{ji} \;\;\; \forall (i, j) \in MV \wedge (j, i) \in EV. \tag{15}$$

The following steps of our procedure will be as described above but restricted to the learning of missing values above the leading diagonal of the incomplete fuzzy preference relation, i.e. p_{ij} with $i < j$. The last step of each iteration will consist in the computation of the corresponding missing values p_{ji} below the leading diagonal again using the reciprocity property.

4 Illustrative example

In this section we use a simple but illustrative example to show the iterative procedure for learning missing values in incomplete fuzzy preference relations.

Let us suppose that an expert provides the following incomplete fuzzy preference relation

$$P = \begin{pmatrix} - & x & 0.4 & x \\ x & - & 0.7 & 0.85 \\ x & 0.4 & - & 0.75 \\ 0.3 & x & x & - \end{pmatrix}$$

The first thing to do is to calculate the consistency level of P, CL_P. To do this, we start calculating all possible εp_{ik}. In this case, we can only calculate εp_{24} and εp_{34} as in the rest of the cases p_{ik} is missing and there is no p_{ij}, p_{jk} to calculate the corresponding cp_{ik}^{j}.

$$\varepsilon p_{24} = |p_{23} + p_{34} - 0.5 - p_{24}| = |0.7 + 0.75 - 0.5 - 0.85| = 0.1$$

$$\varepsilon p_{34} = |p_{32} + p_{24} - 0.5 - p_{34}| = |0.4 + 0.85 - 0.5 - 0.75| = 0$$

These low values of εp_{24} and εp_{34} mean that the inconsistency between p_{24} and the rest of the given information is low while the consistency of p_{34} and the rest of the given information is total.

The next step consists in calculating CL_P as the average of all the εp_{ik} values:

$$CL_P = \frac{\varepsilon p_{24} + \varepsilon p_{34}}{2} = 0.05$$

At this point, we apply our iterative procedure:

$$LMV_1 = \{(1,2), (1,4), (2,1), (3,1), (4,3)\}$$

For each element $(i,k) \in LMV_1$ we calculate cp'_{ik}. For example, cp'_{12} is obtained as:

$$cp'_{12} = \frac{\varepsilon p_{13} + \varepsilon p_{32} - 0.5}{1} = 0.4 + 0.4 - 0.5 = 0.3$$

Using the same procedure we obtain:

$$cp'_{14} = 0.65; \quad cp'_{21} = 0.65; \quad cp'_{31} = 0.55; \quad cp'_{43} = 0.2$$

Next, we proceed to add to each one of the above values a random value $z \in [-0.05, 0.05]$ in order to maintain the expert's level of consistency. As a result of this, we obtain the following incomplete fuzzy preference relation:

$$P = \begin{pmatrix} - & 0.32 & 0.4 & 0.61 \\ 0.68 & - & 0.7 & 0.85 \\ 0.5 & 0.4 & - & 0.75 \\ 0.3 & x & 0.24 & - \end{pmatrix}$$

In the second iteration of our procedure we have $LMV_2 = \{(4,2)\}$,

$$cp'_{42} = \frac{(p_{41} + p_{12} - 0.5) + (p_{43} + p_{32} - 0.5)}{2} = 0.13$$

and $p_{42} = 0.13 + z$ with $z \in [-0.05, 0.05]$ chosen randomly, which gives us:

$$P = \begin{pmatrix} - & 0.32 & 0.4 & 0.61 \\ 0.68 & - & 0.7 & 0.85 \\ 0.5 & 0.4 & - & 0.75 \\ 0.3 & 0.17 & 0.24 & - \end{pmatrix}$$

Obviously, $LMV_3 = \emptyset$ which means that our procedure was successful in the process of discovering all the missing values of the original incomplete fuzzy preference relation P.

5 Concluding remarks and future research

In this paper we have discussed the importance of consistency in decision-making problems, and we have presented a common issue that must be addressed when attempting to solve this kind of problem: incompleteness of information.

In particular, we have focused our attention on incomplete fuzzy preference relations and the issue of finding out their missing values. To do this, we have presented a new iterative procedure to learn missing values which is guided by the additive consistency level of the information known.

In future research, a new induced OWA (IOWA) operator will be developed to aggregate information giving more importance to those experts whose fuzzy preference relations are most consistent. Finally, a general decision procedure, implementing both the learning procedure and the new IOWA operator, will be developed to solve group decision-making problems with incomplete information and inconsistency in the sources of information.

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