A minimum cost consensus-based failure mode and effect analysis framework considering experts’ limited compromise and tolerance behaviors

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Abstract — This study proposes a minimum cost consensus-based failure mode and effect analysis (MCC-FMEA) framework considering experts’ limited compromise and tolerance behaviors, where the first behavior indicates that an FMEA expert might not tolerate modifying his/her risk assessment without limitations, and the second behavior indicates that an FMEA expert will accept risk assessment suggestions without being paid for any cost if the suggested risk assessments fall within his/her tolerance threshold. First, an MCC-FMEA with limited compromise behaviors is presented. Second, experts’ tolerance behaviors are added to the MCC-FMEA with limited compromise behaviors. Theoretical results indicate that in some cases, this MCC-FMEA with limited compromise and tolerance behaviors has no solution. Thus, a minimum compromise adjustment consensus model and a maximum consensus model with limited compromise behaviors are developed and analyzed, and an interactive MCC-FMEA framework, resulting in a FMEA problem consensual collective solution, is designed. A case study, regarding the assessment of COVID-19-related risk in radiation oncology, and a detailed sensitivity and comparative analysis with existing FMEA approaches are provided to verify the effectiveness of the proposed approach to FMEA consensus-reaching.

Index Terms — Reliability management, failure mode and effect analysis, minimum cost consensus model, limited compromise and tolerance behaviors, optimization

I. INTRODUCTION

Failure mode and effect analysis (FMEA) is a progressive reliability management tool [4, 17] widely utilized to identify the failure modes (FMs) of systems, products, or processes and to analyze their possible impacts to take necessary measures to reduce the occurrence of risks. In the past decades, FMEA has been successfully applied in different industrial areas, such as medicine, mechanics, aerospace and healthcare [5, 6, 9, 16].

Ascertaining the risk order of FMs is key in implementing FMEA. This order is often obtained using the concept of the risk priority number (RPN). A FM, which is defined as the product of its occurrence (O), severity (S), and detection (D) risk factors (RFs): $RPN = O \times S \times D$. Accurate values between 1 and 10 are usually used to characterize risk assessment information in the RPN-based FMEA approach [36], with higher numerical point values signifying higher risk levels. The risk ordering of FMs is produced according to their RPN values, which allows for a series of measures to be subsequently carried out to eliminate FMs with high risk levels. A detailed description of the FMEA implementation process is available in other studies [25, 36].

Although the RPN-based FMEA method is very valuable for solving reliability management problems, it is not free of criticism (see [4, 15]). On the one hand, the RPN-based FMEA-precise numerical modeling of the risk assessment information (that is, 1-10 numerical point scale) is not appropriate when the complexity of the FMEA problem leads to uncertain and ambiguous risk assessment information [7, 25, 28]. On the other hand, the RFs are assumed to be equally important, which may not be the case, leading to questions about the validity of the RPN definition mentioned in the previous paragraph [4, 17, 26].

As a response to some of the criticisms, some of the advances to improve the FMEA approach are mentioned here: Bradley and Guerrero [4] proposed a data-elicitation technique-based method to rank FMs; Wang et al. [41] designed an extended generalized TODIM (an acronym in Portuguese of interactive and multicriteria decision making) based FMEA that considers risk indicators’ interaction; Zhang et al. [52] proposed a personalized individual semantics-based method to handle linguistic risk assessments in FMEA with incomplete preference information; Filz et al. [14] presented a data-driven FMEA method based on deep learning approaches; Wang et al. [42] combined several multi-criteria decision making models with linguistic distribution assessment (LDA) to implement the risk assessment of FMs; Liu et al. [27] designed an integrated FMEA approach based on the cloud model theory and the hierarchical TOPSIS (technique for order of preference by similarity to ideal solution) method. Additional improved FMEA approaches can be found in several studies [5, 20, 32, 48, 53]. Liu et al. [26], Spreafico et al. [35] and Huang et al. [22] systematically summarized the progress of the improved FMEA approaches.

Although a lot of advancements on FMEA have been made, the following risk conditions still need better addressing to cope with practical scenarios of FMEA problems:

1) In practical FMEA problems, risk analysts often present different risk assessments due to their experience, cultural, educational backgrounds, and cognitive level differences. However, to the best of our knowledge, almost all existent FMEA models failed to consider the consensus issue among risk analysts. Thus, it is imperative to use a consensus-reaching process in the FMEA
implementation process to help risk analysts achieve a consensus over their risk assessments.

(2) Risk assessments-modifications incur cost, but the resource for consensus-reaching in the FMEA implementation process is often limited. The consensus issue has seldom been considered in the FMEA, let alone a cost-effective consensus-reaching mechanism for supporting conflict-solving in the FMEA. Therefore, it is important to design a consensus building model from the perspective of the consensus cost to improve the quality of the FMEA.

(3) Recently, Cheng et al. [8] proposed a consensus-based group decision making (GDM) approach based on maximum satisfaction under budget constraints for considering decision makers’ limited compromise and tolerance behaviors. As the implementation process of the FMEA can be seen as a special GDM (e.g., [4, 17]), FMEA experts may also exhibit limited compromise and tolerance behaviors in this process. Inspired by Cheng et al. [8], limited compromise and tolerance behaviors under the FMEA framework can be defined: limited compromise behavior means that FMEA experts are prepared to modify their risk assessments within certain boundaries or thresholds, while tolerance behavior refers to an FMEA expert’s willingness to accept risk assessment suggestions, without being compensated (cost of change), when these fall within his/her tolerance threshold. However, the impact of these two behaviors on the FMEA quality has not been modeled by existing FMEA methods. Therefore, it is worth considering these two behaviors of FMEA experts towards the provision of a real-world FMEA problem framework.

Many consensus-reaching models in the area of GDM have been reported to address a decision problem involving multiple experts [10, 33, 34, 40, 43-45, 55]. There are two types of consensus-reaching models, which depend on the implementation of different consensus rules [51]. The first ones are known as consensus-reaching models with identification-direction consensus rules (see [18]), while the second ones are called consensus-reaching models with optimization-based consensus rules (see [30, 37, 46, 50, 54]). Zhang et al. [51] provided a comprehensive comparison analysis of different consensus-reaching models, which shows that optimization-based consensus-reaching models are more consensus efficient than identification-direction consensus-reaching models in the sense of having less information loss.

Motivated by the challenges of addressing the limited compromise and tolerance behaviors of FMEA experts and inspired and enlightened by the advances achieved in optimization-based consensus-reaching processes, this study proposes a minimum cost consensus-based FMEA (MCC-FMEA) framework for reliability management with conflicting risk assessments, with the following contributions:

(1) A minimum cost consensus-reaching process is integrated into the FMEA implementation process, which provides the following advantages: a highly acceptable risk analysis result can be obtained, and the communication and relationship among FMEA experts can be enhanced. A detailed analysis regarding the advantages of using consensus as a decision tool has been discussed by Susskind et al. [38].

(2) Inspired by Cheng et al. [8], the MCC-FMEA framework considers the limited compromise and tolerance behaviors, and a series of optimization-based models are built to model and address these two behaviors. By considering these two behaviors, this study provides a flexible FMEA framework to form an approximate reliability management tool for real-world FMEA problems.

(3) Given the complexity of FMEA problems, multiple rounds of discussions may be required for experts to arrive at a final risk analysis result. Therefore, an interactive MCC-FMEA framework is designed to obtain a highly acceptable collective solution to the FMEA problem based on optimization-based models.

(4) To show the effectiveness of the proposed MCC-FMEA framework, a case study regarding the assessment of COVID-19-related risk in radiation oncology, and a detailed sensitivity and comparative analysis with extant FMEA methods are implemented.

The rest of this paper is structured as below. Section II presents the necessary preliminaries to provide the basis of this study. Section III presents the MCC-FMEA with limited compromise and tolerance behaviors. Section IV provides a case study of the assessment of COVID-19-related risk in radiation oncology to illustrate the use of the MCC-FMEA. Sensitivity and comparative analyses are designed to analyze the MCC-FMEA with limited compromise and tolerance behaviors in Section V. Finally, the conclusions and possible future avenues of research are offered in Section VI.

II. PRELIMINARIES

The basic required knowledge regarding the linguistic assessment models, including the two-tuple linguistic model and the LDA model, and the minimum adjustment/cost consensus model are introduced in this section.

A. Linguistic assessment models

(1) Two-tuple linguistic model

Let \( L = \{l_i, ..., l_p\} \) represent a set linguistic terms with \( g + 1 \) (granularity) terms. Let \( G = \{0, 1, ..., g\} \). The two-tuple linguistic model presented by Herrera and Martínez [19] is introduced below.

Definition 1 [19]. Let \( L \) be as above. The linguistic two-tuple that signifies the equivalent assessment information to \( \tau \in [0, g] \) can be yielded via the following one-to-one mapping function:

\[
\Delta: [0, g] \rightarrow L \times [-0.5, 0.5],
\]

\[
\Delta(\tau) = (l_i, \eta), \quad \text{with } \begin{cases} 
\{l_i, \eta = \text{round}(\tau) \} \\
\eta = \tau - \text{floor}(\tau), \quad \eta \in [-0.5, 0.5]
\end{cases}
\]

Let \( \Delta^{-1} \) be the inverse function of \( \Delta : \Delta^{-1}(l_i, \eta) = i + \eta \). Let \( (l_i, \eta) \) and \( (l_j, \eta_j) \) represent two linguistic two-tuples. Then, it is assumed that \( (l_i, \eta_i) \) is larger than \( (l_j, \eta_j) \) when \( \Delta^{-1}(l_i, \eta_i) > \Delta^{-1}(l_j, \eta_j) \).

Definition 2 [19]. Let \( \{(v_1, \eta_1), ..., (v_t, \eta_t)\} \) denote a set of linguistic two-tuples, where \( v_i \in L \) and \( \eta_i \in [-0.5, 0.5] \). The two-tuple weighted average (TWA) operator over \( \{(v_1, \eta_1), ..., (v_t, \eta_t)\} \) is given as:

\[
TWA_{\pi}(v_1, \eta_1, ..., v_t, \eta_t) = \Delta \sum_{i=1}^{T} \pi_i \Delta^{-1}(v_i, \eta_i),
\]

where \( \pi = (\pi_1, \pi_2, ..., \pi_t)^T \) represents a weighting vector, \( \pi_i > 0 \), and \( \sum_{i=1}^{T} \pi_i = 1 \).

(2) Linguistic distribution assessment model
The LDA model [24, 49], where symbolic proportions are distributed to all linguistic terms in $L$, is introduced below.

**Definition 3.** Let $L$ be as above. A distribution assessment of $L$ is denoted by $A = \{ (l_i, \rho_s) | s = 0, 1, ..., g \}$, where $\rho_s \in [0, 1]$ is the symbolic proportion distributed to linguistic term $l_i$ subject to the constraint
\[
\sum_{s=0}^{g} \rho_s = 1.
\]

The set of LDAs defined on the linguistic term set $L$ is represented as $LD(L)$.

**Definition 4.** Let $A = \{ (l_i, \rho_s) | s = 0, 1, ..., g \}$ be as above. The expectation of $A$ is computed by:
\[
E(A) = \Delta \sum_{s=0}^{g} \rho_s \times NS(l_i)
\]
where $NS(l_i)$ denotes the numerical scale of $l_i \in L$. In this study, it is set that $NS(l_i) = s$.

Let $A_t = \{ (l_i, \rho'_{s}) | s = 0, 1, ..., g \}$ and $A_c = \{ (l_i, \rho^c_{s}) | s = 0, 1, ..., g \}$ be two LDAs defined on $L$. Then, we have that: (i) if $E(A_t) < E(A_c)$, $A_t$ is smaller than $A_c$; and (ii) if $E(A_t) = E(A_c)$, $A_t$ is equivalent to $A_c$.

**Definition 5.** Let $A_t$ and $A_c$ be as above. The distance between $A_t$ and $A_c$ is calculated as below:
\[
d(A_t, A_c) = \Delta \sum_{s=0}^{g} \rho'_s \times NS(l_i) - \Delta \sum_{s=0}^{g} \rho^c_s \times NS(l_i) / g
\]
Clearly, $d(A_t, A_c) \in [0, 1]$.

**Definition 6.** Let $\{ A_1, A_2, ..., A_T \}$ be a set of LDAs defined on $L$:
\[
A_i = \{ (l_i, \rho'_s) | s = 0, 1, ..., g \} \text{ (} i = 1, 2, ..., T \text{)}.
\]
The weighted averaging operator over $\{ A_1, A_2, ..., A_T \}$ is:
\[
LDAW(\{ A_1, A_2, ..., A_T \}) = \{ (l_i, \rho^r_s) | s = 0, 1, ..., g \}
\]
where $\rho^r_s = \sum_{t=1}^{T} \pi_t \times \rho'_s$.

**B. Minimum adjustment or cost consensus model**

In recent years, some consensus-reaching models with minimum adjustment or cost have been proposed to improve consensus efficiency with respect to information loss or consensus cost. Dong et al. [13] designed the following minimum adjustment consensus model:
\[
\min \sum_{i=1}^{n} d(r^k, \bar{r}^k)
\]
\[
s.t. \begin{cases} 
\bar{r} = F(r^1, ..., r^m) \\
CL(\bar{r}, \bar{r}) \geq \theta, \text{ } k = 1, ..., m
\end{cases}
\]
where $\{ r^1, ..., r^m \}$ and $\{ \bar{r}^1, ..., \bar{r}^m \}$ represent the sets of original and adjusted assessments, respectively; $d(r^k, \bar{r}^k)$ denotes the distance between assessments $r^k$ and $\bar{r}^k$. The first constraint of Model (7) fuses individual adjusted assessments $\{ \bar{r}^1, ..., \bar{r}^m \}$ into a collective adjusted assessment $\bar{r}$ utilizing an aggregation function $F$. The second constraint ensures that the consensus level of individual $e_i$ regarding his/her adjusted assessment $\bar{r}^k$ exceeds the threshold $\theta$.

Notably, changing the objective function of Model (7) to \[
\min \sum_{i=1}^{n} c^k \cdot d(r^k, \bar{r}^k),
\]
where $c^k$ denotes the unit consensus cost of individual $e_i$ ($k=1, 2, ..., m$), generates the minimum cost consensus model (MCCM) [50]. Moreover, removing the aggregation function from the MCCM [50] generates Ben-Arie and Easton’s MCCM [1]. However, the group assessment is the average of the largest and smallest extreme assessments when using Ben-Arie and Easton’s MCCM [1], which is not in line with realistic GDM problems. For a comprehensive description of the consensus-reaching models with minimum adjustment or cost, please check the study by Zhang et al. [54].

**III. MINIMUM COST CONSENSUS-BASED FMEA FRAMEWORK WITH LIMITED COMPROMISE AND TOLERANCE BEHAVIORS**

The MCC-FMEA framework with a detailed resolution procedure is designed in this section. Moreover, the MCCM with limited compromise and tolerance behaviors within the MCC-FMEA framework is developed.

**A. The proposed MCC-FMEA framework**

As analyzed in Section I, obtaining a FMEA consensual decision is an important issue in practice. Meanwhile, the limited compromise and tolerance behaviors are essential elements that cannot be ignored in FMEA consensus-reaching. Inspired by the optimization-based consensus-reaching models (e.g., [2, 23, 54]), an MCC-FMEA framework with limited compromise and tolerance behaviors is proposed, which is shown in Steps 1–12 of Fig. 1. Next, the details of each step in the MCC-FMEA framework are described.

Step 1: *Assemble the FMEA team.* To achieve a comprehensive risk analysis, the FMEA implementation process often involves multiple experts, $E = \{ e_1, e_2, ..., e_n \}$. The weighting vector of $E$ is normalized $\lambda = (\lambda_1, \lambda_2, ..., \lambda_q)^T$, where $\lambda_q \in [0, 1]$ is the normalized weight associated with FMEA expert $e_q$, that is $\sum_{q=1}^{n} \lambda_q = 1$.

Step 2: *Analyze the risk analysis object (system, product, process, or service)* in which FMEA will be implemented. Prior to applying the FMEA, the FMEA team members must carefully analyze all procedures of the risk analysis object.

Step 3: *Ascertain the potential FMs associated with the risk analysis object.* It is assumed that $p$ FMs are ascertained by the FMEA team, which are denoted by $FM = \{ FM_1, ..., FM_p \}$.

Step 4: *Determine the weights of $O, S,$ and $D.*$ Let $RF = \{ RF_1, ..., RF_q \}$ be the set of $q$ RFs used as the criteria for evaluating FMs. Let $w = \{ w_1, ..., w_q \}$ denote weight vector over $RF$, where $w_i \geq 0$ ($i=1, 2, ..., q$) represents the weight of $RF_i$, that is $\sum_{i=1}^{q} w_i = 1$. In this study, we assumed that three RFs, O, S, and D, are employed, and their weights are directly offered by $E$.

Step 5: *Ascertain the information format for assessing risk level of FMs over RFs.* The LDA is used as the information format to present the assessments of FMEA experts because it is more consistent with the expression habits of risk analysis experts compared with the 1-10 numerical point scale and different linguistic information formats [21].

Step 6-8: *Analyze the effects and causes of* $\{ FM_1, ..., FM_p \}$.

Identify the control and/or prevention measure associated with each cause.

Step 9: *Assess the risk level of FMs over RFs via LDA.* As analyzed in Step 5, the LDA model is an effective information format to convey risk assessments under complex situations. Concretely, the linguistic distribution risk assessment matrix (LDARM) given by FMEA experts is denoted as $R^i = (r^i_{pj})_{p,q}$.
(k = 1, 2, ..., m), where \( r^f_s = [(l, P^f_s)] | s = 0,1,...,g \) signifies the assessment of \( FM_s \) with respect to \( R^f \).

Step 10: Utilize the MCCM with limited compromise and tolerance behaviors to produce a consensual collective LDRAM of FMs with respect to RFs (O, D, and S). The obtained consensual collective LDRAM is denoted as \( R^c = (r^c_{pq}) \). Concretely, three procedures are involved in this step: (a) Risk assessment fusion: individual LDRAMs are aggregated into a collective LDRAM over FMs and RFs. (b) Consensus measurement: the current consensus level among FMEA experts is calculated. (c) Obtaining consensual risk assessments among FMEA experts: the MCCM with limited compromise and tolerance behaviors is proposed to help FMEA experts obtain the consensual collective LDRAM, and this is done by minimizing the linguistic assessments adjustment cost. Detailed information regarding this consensus process is proposed in Section III.B.

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**Fig. 1.** The MCCM-based FMEA framework considering the limited compromise and tolerance behaviors

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**Step 11: Generate the risk order of FMs.** Based on the consensual collective LDRAM, the overall risk evaluation value for \( FM_s \), \( EV_s \), is obtained:

\[
EV_s = LDWA_s(r^s_1, r^s_2, ..., r^s_g) = [(l, P^s)] | s = 0,1,...,g
\]

where \( P^s = \sum_{j=1}^{g} w_j \times \rho^s_{j} \). Based on \( [EV_1, ..., EV_m] \), the FMs \( [FM_1, ..., FM_m] \) risk order is derived.

**Step 12: Recommend corrective actions and modifications.** According to the risk order of FMs generated using Step 11, the precautions and corrective actions to eliminate the potential risk of FMs with high risk levels are determined.

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**B. Minimum cost consensus model with limited compromise and tolerance behaviors**

In this section, we present the MCCM with limited compromise and tolerance behaviors to support FMEA experts in achieving a consensus regarding their risk assessments.

**B.1. Risk assessment fusion and consensus measure**

The LDAWA operator generates the collective LDRAM \( R^c = (r^c_{pq}) \) from individual LDRAMs \( R^p = (r^p_{pq}) \),

\[
r^c_{pq} = [(l, P^c_{pq})] | s = 0,1,...,g \) = LDWA_s(r^s_{pq}) (9)
\]

where \( P^c_{pq} = \sum_{k=1}^{m} \lambda_k \times P^k_{pq} \).

The consensus level of FMEA experts measures in (0,1) the similarity between their corresponding individual and collective LDRAMs [53]:

\[
CL(R^c, R^p) = 1 - d(R^c, R^p) = 1 - \frac{1}{pqg} \sum_{i=1}^{p} \sum_{j=1}^{q} |x_i^c - x_j^c| (10)
\]

where \( x_i^c = \Delta^c(E(r^c_{pq})) \) and \( x_j^p = \Delta^c(E(r^p_{pq})) \).

The collective consensus level is defined as the average of FMEA experts’ individual consensus levels [53]:

\[
CL(R^c, ..., R^m) = \frac{1}{m} \sum_{i=1}^{m} CL(R^c, R^i) = 1 - \frac{1}{mpqg} \sum_{i=1}^{m} \sum_{j=1}^{p} \sum_{k=1}^{q} |x_i^c - x_j^i| (11)
\]

A larger value of \( CL(R^c, ..., R^m) \) indicates a higher level of consensus on the collective LDRAM \( R^c \). In general, it is argued that the consensus among FMEA experts is reached when \( CL(R^c, ..., R^m) \geq \theta \), where \( \theta \in [0,1] \) denotes a minimum consensus threshold.

**B.2. The presentation of the MCCM**

The MCCM with limited compromise behaviors is first presented. Then, the tolerance behaviors are considered in the proposed MCCM. Finally, desirable properties of the MCCM with limited compromise and tolerance behaviors are discussed.

**I) The MCCM with limited compromise behaviors**

To reach a consensus, FMEA experts need to modify their LDRAMs. Let \( \tilde{R}^i = (r^i_{pq}) \) be the adjusted LDRAM associated with \( R^i = (r^i_{pq}) \), where \( r^i_{pq} = [(l, P^i_{pq})] | s = 0,1,...,g \). The change from \( R^i \) to \( \tilde{R}^i \) means cost, and the available cost is often limited. Thus, the adjustment cost from \( [R^1, R^2, ..., R^m] \) to \( [\tilde{R}^1, \tilde{R}^2, ..., \tilde{R}^m] \) is to be optimized:

\[
\min \sum_{k=1}^{m} \sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij} \times d(\tilde{r}^i_{pq}, \tilde{r}^i_{pq}) (12)
\]

where \( d(\tilde{r}^i_{pq}, \tilde{r}^i_{pq}) = |x_i^c - x_j^i|/g \) and \( x_i^c = \Delta^c(E(r^c_{pq})) \), \( x_j^i = \Delta^c(E(r^i_{pq})) \).

Meanwhile, the consensus level among the adjusted LDRAMs \( [\tilde{R}^1, \tilde{R}^2, ..., \tilde{R}^m] \) must meet the consensus threshold requirement:

\[
CL(\tilde{R}^1, \tilde{R}^2, ..., \tilde{R}^m) \geq \theta \). (13)
\]

FMEA experts \( \epsilon^i \) \((k \in M)\) will present limited compromise behaviors in modifying \( r^i_{pq} \) to \( \tilde{r}^i_{pq} \), which means they will not tolerate adjusting their risk assessments from \( r^i_{pq} \) to \( \tilde{r}^i_{pq} \) without limitations. In other words, FMEA experts \( \epsilon^i \) \((k \in M)\) will accept \( \tilde{r}^i_{pq} \) only if it is within their compromise boundary of \( r^i_{pq} \), that is when
where \( d(r_i^k, r_j^k) \leq \beta_i \forall k \in M, i \in P, j \in Q \),
where \( M = \{1, \ldots, m\}, P = \{1, \ldots, p\}, Q = \{1, \ldots, q\} \), and \( \beta_i \geq 0 \) is the compromise threshold of FMEA expert \( e_i \).

Based on the above analysis, the MCCM with limited compromise behaviors becomes:

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{p} \sum_{k=1}^{q} c_i \cdot d(r_i^k, r_j^k)
\]

subject to:

\[
d(r_i^k, r_j^k) \leq \beta_i \quad k \in M; \quad i \in P; \quad j \in Q \quad (a)
\]

\[
C(r_i^k, r_j^k) = \begin{cases} 
0, & d(r_i^k, r_j^k) \leq \alpha_i \\
\varepsilon \cdot d(r_i^k, r_j^k), & (d(r_i^k, r_j^k) > \alpha_i) 
\end{cases} 
\]

subject to:

\[
d(r_i^k, r_j^k) \leq \beta_i \quad k \in M; \quad i \in P; \quad j \in Q \\
C(r_i^k, r_j^k) \geq \theta \\
F_i^k = LDAW(L_1, \ldots, L_m), \quad k \in M; \quad i \in P; \quad j \in Q \\
F_i^k, r_j^k \in LD(L), \quad k \in M; \quad i \in P; \quad j \in Q 
\]

\[(b) \quad \] (c) \quad \] (d)

(2) The MCCM with limited compromise and tolerance behaviors

In addition to limited compromise behaviors, FMEA experts \( e_i \) \( k \in M \) will also present limited tolerance behaviors, which means when the suggested assessment falls within their tolerance threshold, \( \alpha_i \geq 0 \) such that \( \alpha_i \leq \beta_i \). They will be accepted without being reaped for their corresponding adjusting costs. Considering the tolerance behaviors, the cost of adjusting \( r_i^k \) to \( r_j^k \) is defined as follows:

\[
C(r_i^k, r_j^k) = \begin{cases} 
0, & d(r_i^k, r_j^k) \leq \alpha_i \\
\varepsilon \cdot d(r_i^k, r_j^k), & (d(r_i^k, r_j^k) > \alpha_i) 
\end{cases} 
\]

where the first line indicates that no cost is required when the deviation between \( r_i^k \) and \( r_j^k \) is less than or equal to \( \alpha_i \), and the second line means that the cost is \( \varepsilon \cdot d(r_i^k, r_j^k) \) when the deviation is larger than \( \alpha_i \).

So, the MCCM with limited compromise and tolerance behaviors becomes:

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{p} \sum_{k=1}^{q} C(r_i^k, r_j^k)
\]

subject to:

\[
d(r_i^k, r_j^k) \leq \beta_i \\
C(r_i^k, r_j^k) \geq \theta \\
F_i^k = LDAW(L_1, \ldots, L_m), \quad i \in P; \quad j \in Q \\
F_i^k, r_j^k \in LD(L), \quad k \in M; \quad i \in P; \quad j \in Q 
\]

Specifically, Model (17) is expressed as follows:

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{p} \sum_{k=1}^{q} C(r_i^k, r_j^k)
\]

subject to:

\[
|y_i^k - x_i^k| \leq \alpha_i \\
|y_i^k - x_i^k| \leq \beta_i, \quad k \in M; \quad i \in P; \quad j \in Q \\
C(r_i^k, r_j^k) \geq \theta \\
F_i^k = LDAW(L_1, \ldots, L_m), \quad i \in P; \quad j \in Q \\
F_i^k, r_j^k \in LD(L), \quad k \in M; \quad i \in P; \quad j \in Q
\]

\[(e) \quad \] (f) \quad \] (g)

where \( x_i^k = \sum_{s=1}^{m} \alpha_i' \cdot NS(l_s) \), and with decision variables \( p_i^k \) and \( \overline{p}_{j}^k \) \( k \in M, i \in P, j \in Q, s \in G \). Propositions 1-3 allow to transform Model (18) into a 0-1 mixed linear programming model.

**Proposition 1:** By introducing a set of 0-1 variables \( y_i^k \in \{0, 1\} \), a new optimization-based model is constructed:

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{p} \sum_{k=1}^{q} C(r_i^k, r_j^k)
\]

subject to:

\[
|y_i^k - x_i^k| \leq \alpha_i, \quad k \in M; \quad i \in P; \quad j \in Q \\
|y_i^k - x_i^k| \leq \beta_i, \quad k \in M; \quad i \in P; \quad j \in Q \\
C(r_i^k, r_j^k) \geq \theta \\
F_i^k = LDAW(L_1, \ldots, L_m), \quad i \in P; \quad j \in Q \\
F_i^k, r_j^k \in LD(L), \quad k \in M; \quad i \in P; \quad j \in Q \\
y_i^k \in \{0, 1\}, \quad k \in M; \quad i \in P; \quad j \in Q; \quad s \in G
\]

All other constraints in model (18)

Then, we have that Model (19) is equivalent to Model (18).

**Proof:** Constraints (b) and (c) ensure that (i) \( y_i^k = 0 \) or 1 when \( |x_i^k - x_j^k| / g \leq \alpha_i \), and (ii) \( y_i^k = 1 \) when \( (x_i^k - x_j^k) / g > \alpha_i \).

Let \( o_1 = \{(k, i, j) | (x_i^k - x_j^k) / g \leq \alpha_i, \quad k \in M, \quad i \in P, \quad j \in Q\} \) and \( o_2 = \{(k, i, j) | (x_i^k - x_j^k) / g > \alpha_i, \quad k \in M, \quad i \in P, \quad j \in Q\} \). Then the objective function of Model (19) can be written as below:

\[
\sum_{i=1}^{n} \sum_{j=1}^{p} \sum_{k=1}^{q} C(r_i^k, r_j^k) = \sum_{(k, i, j) \in o_1} c_i \cdot y_i^k \cdot |x_i^k - x_j^k| / g + \sum_{(k, i, j) \in o_2} c_i \cdot y_i^k \cdot |x_i^k - x_j^k| / g 
\]

It achieves optimal value only when \( y_i^k = 0 \) for \( (k, i, j) \in o_1 \) because we aim to obtain its minimum value.

Thus, in Model (19), we have that: \( C(r_i^k, r_j^k) = 0 \) when \( |x_i^k - x_j^k| / g \leq \alpha_i \), while \( C(r_i^k, r_j^k) = c_i \cdot |x_i^k - x_j^k| / g \) when \( |x_i^k - x_j^k| / g > \alpha_i \). This conclusion is consistent with the constraint (a) of Model (18).

So, Model (18) is equivalent to Model (19). This completes the proof of Proposition 1.

The following results by [3] is recalled:

**Proposition 2** [3]: In a mixed 0-1 programming model, the term \( \sum_{i=1}^{n} a_i y_i \), where \( x \) is a binary variable and \( y_i \) are variables with finite bounds, can be replaced via a new variable \( z \) and the following linear constraints:

\[
xy^k \leq z \leq xy^k \\
C(r_i^k, r_j^k) \geq \theta \\
F_i^k = LDAW(L_1, \ldots, L_m), \quad i \in P; \quad j \in Q \\
F_i^k, r_j^k \in LD(L), \quad k \in M; \quad i \in P; \quad j \in Q
\]

Specifically, Model (17) is expressed as follows:

where \( y_i^k = \sum_{s=1}^{m} p_i^k \cdot NS(l_s) \), and with decision variables \( p_i^k \) and \( \overline{p}_{j}^k \) \( k \in M, i \in P, j \in Q, s \in G \). Propositions 1-3 allow to transform Model (18) into a 0-1 mixed linear programming model.

**Proposition 3** Model (19) is equivalent to the following 0-1 mixed linear programming model:

The following results by [3] is recalled:

**Proposition 2** [3]: In a mixed 0-1 programming model, the term \( \sum_{i=1}^{n} a_i y_i \), where \( x \) is a binary variable and \( y_i \) are variables with finite bounds, can be replaced via a new variable \( z \) and the following linear constraints:

\[
\sum_{i=1}^{n} a_i y_i \leq z \leq \sum_{i=1}^{n} a_i y_i \\
C(r_i^k, r_j^k) \geq \theta \\
F_i^k = LDAW(L_1, \ldots, L_m), \quad i \in P; \quad j \in Q \\
F_i^k, r_j^k \in LD(L), \quad k \in M; \quad i \in P; \quad j \in Q
\]

Specifically, Model (17) is expressed as follows:

where \( y_i^k = \sum_{s=1}^{m} p_i^k \cdot NS(l_s) \), and with decision variables \( p_i^k \) and \( \overline{p}_{j}^k \) \( k \in M, i \in P, j \in Q, s \in G \). Propositions 1-3 allow to transform Model (18) into a 0-1 mixed linear programming model.
min \sum_{k=1}^{m} \sum_{i=1}^{p} \sum_{j=1}^{q} c^k \cdot z^k_{ij} \\
0 \leq z^k_{ij} \leq l^k_{ij}, \quad k \in M; \ i \in P; \ j \in Q \tag{a} \\
a^k_i - (1-x^k_i) \leq l^k_{ij} \leq a^k_i, \quad k \in M; \ i \in P; \ j \in Q \tag{b} \\
\frac{x^k_i - l^k_{ij}}{g} \leq a^k_i, \quad k \in M; \ i \in P; \ j \in Q \tag{c} \\
\frac{l^k_{ij} + x^k_i}{g} \leq a^k_i, \quad k \in M; \ i \in P; \ j \in Q \tag{d} \\
\frac{x^k_i - l^k_{ij}}{g} \geq \beta_i, \quad k \in M; \ i \in P; \ j \in Q \tag{e} \\
\frac{l^k_{ij} + x^k_i}{g} \geq \beta_i, \quad k \in M; \ i \in P; \ j \in Q \tag{f} \\
\frac{x^k_i - l^k_{ij}}{g} \leq \beta_i, \quad k \in M; \ i \in P; \ j \in Q \tag{g} \\
\frac{l^k_{ij} + x^k_i}{g} \leq \beta_i, \quad k \in M; \ i \in P; \ j \in Q \tag{h} \\
\text{The optimal values to } a^k_i \text{ (} k \in M; \ i \in P; \ j \in Q \text{)} \text{ are obtained by solving the above problem.} \\
\text{The optimal objective function value is denoted as } C^* \text{. Then, it is } C^* \leq C^2. \tag{22} \\
\text{Proof: Let } \{r^k_i\} \text{ be the optimal values to decision variables } \{r^k_i\} \text{ of Model } P_2 \text{. So,} \\
C^* = \sum_{k=1}^{m} \sum_{i=1}^{p} \sum_{j=1}^{q} c^k \cdot d(r^k_i, r^k_i). \\
\text{Obviously, } \{r^k_i\} \text{ is the feasible solution to Model } P_1 \text{. Let} \\
\alpha_1 = \{(k,j) | d(r^k_i, r^k_j) < a_1, k \in M; i \in P; j \in Q\} \text{ and} \\
\alpha_2 = \{(k,j) | d(r^k_i, r^k_j) > a_2, k \in M; i \in P; j \in Q\} \text{. Under feasible solution } \\
\{r^k_i\} \text{, the objective function of Model } P_1 \text{ is below:} \\
C^1 = \sum_{(k,j) \in \alpha_1} c^k \cdot d(r^k_i, r^k_j) + \sum_{(k,j) \in \alpha_2} c^k \cdot d(r^k_i, r^k_j). \\
\text{So, we have that } C^1 \leq C^* \leq C^2 \text{ for all } k \in M. \\
\text{The proof of Proposition 5 is omitted as it is similar to that of Proposition 4. This is consistent with the meaning of tolerance, that is the lower the tolerant experts are, the lower the consensus cost will be.} 

B.3. The interactive minimum cost consensus-reaching framework 

In some cases, Model (18) has no solution, which is mainly caused by the limited compromise behaviors. To analyze this issue, the following minimum compromise adjustment consensus model (MCACM) is presented:

\min \frac{\sum_{i=1}^{p} \sum_{j=1}^{q} \left| x^k_i - l^k_{ij} \right|}{g} \tag{23} \\
C(L) = \frac{1}{1 - \sum_{i=1}^{p} \sum_{j=1}^{q} \left| x^k_i - l^k_{ij} \right|} \geq \theta \\
\text{The optimal solution of Model (23) is } \left\{ \mu^k_{ij}, \nu^k_{ij} \right\}(k \in M; i \in P; j \in Q) \text{. The optimal objective function value is denoted as } C^* \text{. Then, it is } C^* \leq C^2. \tag{24} \\
\text{Proof: Let } \{r^k_i\} \text{ be the optimal values to decision variables } \{r^k_i\} \text{ of Model } P_2 \text{. So,} \\
C^* = \sum_{k=1}^{m} \sum_{i=1}^{p} \sum_{j=1}^{q} c^k \cdot d(r^k_i, r^k_i). \\
\text{Obviously, } \{r^k_i\} \text{ is the feasible solution to Model } P_1 \text{. Let} \\
\alpha_1 = \{(k,j) | d(r^k_i, r^k_j) < a_1, k \in M; i \in P; j \in Q\} \text{ and} \\
\alpha_2 = \{(k,j) | d(r^k_i, r^k_j) > a_2, k \in M; i \in P; j \in Q\} \text{. Under feasible solution } \\
\{r^k_i\} \text{, the objective function of Model } P_1 \text{ is below:} \\
C^1 = \sum_{(k,j) \in \alpha_1} c^k \cdot d(r^k_i, r^k_j) + \sum_{(k,j) \in \alpha_2} c^k \cdot d(r^k_i, r^k_j). \\
\text{So, we have that } C^1 \leq C^* \leq C^2 \text{ for all } k \in M. \\
\text{The proof of Proposition 5 is omitted as it is similar to that of Proposition 4. This is consistent with the meaning of tolerance, that is the lower the tolerant experts are, the lower the consensus cost will be.} 

Proposition 6: If the optimal function value of Model (23) is equal to zero, that is, } \varepsilon = 0, \text{ then Model (18) has feasible solution.} 

\min \frac{\sum_{i=1}^{p} \sum_{j=1}^{q} \left| x^k_i - l^k_{ij} \right|}{g} \tag{25} \\
C(L) = \frac{1}{1 - \sum_{i=1}^{p} \sum_{j=1}^{q} \left| x^k_i - l^k_{ij} \right|} \geq \theta \\
\text{The optimal solution of Model (25) is } \left\{ \mu^k_{ij}, \nu^k_{ij} \right\}(k \in M; i \in P; j \in Q) \text{. The optimal objective function value is denoted as } C^* \text{. Then, it is } C^* \leq C^2. \\
\text{Proof: Let } \{r^k_i\} \text{ be the optimal values to decision variables } \{r^k_i\} \text{ of Model } P_2 \text{. So,} \\
C^* = \sum_{k=1}^{m} \sum_{i=1}^{p} \sum_{j=1}^{q} c^k \cdot d(r^k_i, r^k_i). \\
\text{Obviously, } \{r^k_i\} \text{ is the feasible solution to Model } P_1 \text{. Let} \\
\alpha_1 = \{(k,j) | d(r^k_i, r^k_j) < a_1, k \in M; i \in P; j \in Q\} \text{ and} \\
\alpha_2 = \{(k,j) | d(r^k_i, r^k_j) > a_2, k \in M; i \in P; j \in Q\} \text{. Under feasible solution } \\
\{r^k_i\} \text{, the objective function of Model } P_1 \text{ is below:} \\
C^1 = \sum_{(k,j) \in \alpha_1} c^k \cdot d(r^k_i, r^k_j) + \sum_{(k,j) \in \alpha_2} c^k \cdot d(r^k_i, r^k_j). \\
\text{So, we have that } C^1 \leq C^* \leq C^2 \text{ for all } k \in M. 

In other words, Model (18) has feasible solution.
To deal with the case of Model (18) being unsolvable, the maximum consensus model with limited compromise behaviors is presented below:

$$\text{max } C L(R^1, ..., R^n)$$

$$s.t.:$$

$$x_i^k - x_i^j \leq \beta_i^k, \quad k \in M; \quad i \in P; \quad j \in Q$$

$$C L(R^1, ..., R^n) = 1 - \frac{1}{mpq \sum_{i=1}^p \sum_{j=1}^q \sum_{k=1}^m |x_i^k - x_j^k|}$$

$$x_i^k = \sum_{e \in E} \alpha_{e} p_{e}^k \cdot N(S(e)), \quad k \in M; \quad i \in P; \quad j \in Q$$

$$p_{e}^k = \sum_{e \in E} \alpha_{e} p_{e}^k \cdot NS(e), \quad i \in P; \quad j \in Q$$

$$\sum_{e \in E} p_{e}^k = 1, \quad k \in M; \quad i \in P; \quad j \in Q$$

$$\sum_{e \in E} p_{e}^k = 0, \quad k \in M; \quad i \in P; \quad j \in Q; \quad s \in G$$

(23)

A similar form of the transformation process of Model (19), detailed above, can be applied to convert Model (23) to a linear programming model, which is omitted.

**Proposition 7**: Let $CL'$ be the optimal value of Model (23)'s objective function. If $CL \geq 0$, then model (18) has solution.

**Proof**: Let $\{\overline{p}_{e}^k, \overline{p}_{e}^k, \overline{x}_{i}^k, \overline{x}_{i}^k\} (k \in M, i \in P, j \in Q)$ be the optimal values of Model (23). Then, $\{\overline{p}_{e}^k, \overline{p}_{e}^k, \overline{x}_{i}^k, \overline{x}_{i}^k\}$ satisfies constraints (b)-(h) of Model (18). In addition, constraint (a) of Model (18) is always feasible in different situations. Thus, Model (18) has feasible solution.

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**IV. CASE ILLUSTRATION**

This section illustrates the use of the MCC-FMEA framework to assess COVID-19-related risks in radiation oncology [39]. The COVID-19 pandemic has posed a huge challenge to the provision of safe and timely care for cancer patients, which is particularly serious for high-risk tumor patients due to their advanced age or their immune system disorders caused by treatments. Radiation oncology (radiotherapy/radiation therapy) is an essential medical specialty of a multidisciplinary approach to cancer treatment. Patients often suffer from extended treatments and are in close contact with staff for positioning, which increases the possibility of nosocomial transmission during the COVID-19 pandemic. To reduce transmission risk for patients and providers, the oncology community undertakes substantial workflow adaptations, and various control measures have been proposed and implemented. It is an extremely important issue to analyze the impacts of control measures on the safety of the radiation oncology workflow and potential for transmission. The MCC-FMEA approach is therefore used to assess the risks of pandemic-associated workflow adaptations during the COVID-19 pandemic. Viscariello et al. [39]
identified a list of 33 FMs in the radiation therapy workflow for patients during the COVID-19 pandemic. In the case study, six high-risk FMs \{FM_1, \ldots, FM_6\} were selected for further analysis. The details of the selected six FMs are shown in Table 2.

Table 2: FMs and their failure causes and effects in the radiation therapy process during COVID-19 pandemic (taken from [39])

<table>
<thead>
<tr>
<th>No.</th>
<th>Failure mode</th>
<th>Failure cause</th>
<th>Failure effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM_1</td>
<td>Infectious person arrives at the department with unknown symptom status</td>
<td>Visual marker for screened status not used (piece of paper, sticker)</td>
<td>Patient or staff exposure from infectious patient</td>
</tr>
<tr>
<td>FM_2</td>
<td>Infectious person not caught at initial screening</td>
<td>Initial hospital and department entry screening not effective, person enters from unsecured entrance</td>
<td>Patient or staff exposure from infectious person</td>
</tr>
<tr>
<td>FM_3</td>
<td>Infectious person not caught at initial screening</td>
<td>Person is unclear about symptoms or mistakes it for a chronic condition, treatment effect, etc.</td>
<td>Patient or staff exposure from infectious person</td>
</tr>
<tr>
<td>FM_4</td>
<td>Waiting/changing room not cleaned routinely</td>
<td>No policy is in place, lack of cleaning supplies or staff</td>
<td>Patient or staff exposure from infectious person</td>
</tr>
<tr>
<td>FM_5</td>
<td>Infectious person not caught at initial screening</td>
<td>Person is asymptomatic</td>
<td>Patient or staff exposure from infectious person</td>
</tr>
<tr>
<td>FM_6</td>
<td>Infectious person arrives at the department with unknown symptom status</td>
<td>Patient or staff is unclear about what constitutes exposure to a positive person</td>
<td>Patient or staff exposure from infectious person</td>
</tr>
</tbody>
</table>

In this case study, O, S, and D are used to evaluate \{FM_1, \ldots, FM_6\}. For convenience, let \(RF_1 = O\), \(RF_2 = S\), and \(RF_3 = D\) and assume their weighting vector is \(w = (1/3,1/3,1/3)^T\). There are four FMEA experts (denoted as \(E = \{e_1, e_2, e_3, e_4\}\), with weighting vector \(\lambda = (0.25, 0.25, 0.25, 0.25)^T\); and the seven-grade linguistic term set (see Table 3) is used by them to provide LDARMs (shown in Tables 4-7).

Table 3: Seven-grade linguistic term set

<table>
<thead>
<tr>
<th>O</th>
<th>S</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i_0)</td>
<td>Very low</td>
<td>Slight</td>
</tr>
<tr>
<td>(i_1)</td>
<td>Low</td>
<td>Sligh</td>
</tr>
<tr>
<td>(i_2)</td>
<td>Reasonably low</td>
<td>Reasonably</td>
</tr>
<tr>
<td>(i_3)</td>
<td>Average</td>
<td>Average</td>
</tr>
<tr>
<td>(i_4)</td>
<td>Reasonably frequent</td>
<td>Reasonably high</td>
</tr>
<tr>
<td>(i_5)</td>
<td>Frequent</td>
<td>High</td>
</tr>
<tr>
<td>(i_6)</td>
<td>Very frequent</td>
<td>Very high</td>
</tr>
</tbody>
</table>

In Table 4, the LDARMs \(R'_1 = (e'_i \cdot p_{ij})_{i,j=1}^6\) are obtained. Herein, we show how to obtain \(\rho_{1,2}^{L'}\) in \(e'_i = (\cup_{s=1}^{6} p_{is}^{L'} R_{ij}^{L'})\) : \(\rho_{1,2}^{L'} = 0.25 \times 0.4 + 0.25 \times 0.4 + 0.25 \times 0.4 + 0.25 \times 0.2 = 0.2\).
Further, we obtain $r_{ij}^* = \{(\ell_i, 0.2), (\ell_i, 0.275), (\ell_i, 0.25), (\ell_i, 0.275)\}$.  

<table>
<thead>
<tr>
<th>FMs</th>
<th>$R_F$</th>
<th>$R_{F^*}$</th>
<th>$R_{F^*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{M1}$</td>
<td>$(\ell_i, 0.2), (\ell_i, 0.275)$, $T$</td>
<td>$(\ell_i, 0.375), (\ell_i, 0.125)$</td>
<td>$(\ell_i, 0.0625), (\ell_i, 0.375), (\ell_i, 0.25), (\ell_i, 0.375)$</td>
</tr>
<tr>
<td>$F_{M2}$</td>
<td>$(\ell_i, 0.05), (\ell_i, 0.1), (\ell_i, 0.15), (\ell_i, 0.225)$</td>
<td>$(\ell_i, 0.375), (\ell_i, 0.225)$</td>
<td>$(\ell_i, 0.05), (\ell_i, 0.05), (\ell_i, 0.15), (\ell_i, 0.225)$</td>
</tr>
<tr>
<td>$F_{M3}$</td>
<td>$(\ell_i, 0.05), (\ell_i, 0.05), (\ell_i, 0.375), (\ell_i, 0.15)$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25)$</td>
<td>$(\ell_i, 0.05), (\ell_i, 0.05), (\ell_i, 0.375), (\ell_i, 0.15)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FMs</th>
<th>$R_F$</th>
<th>$R_F$</th>
<th>$R_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{M1}$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
<td>$(\ell_i, 0.0625), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
<td>$(\ell_i, 0.0625), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
</tr>
<tr>
<td>$F_{M2}$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
</tr>
<tr>
<td>$F_{M3}$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.25), (\ell_i, 0.25), (\ell_i, 0.15)$</td>
</tr>
</tbody>
</table>

The parameters values are set as: $c^1 = 0.6$, $c^2 = 0.9$, $c^3 = 0.4$, $c^4 = 0.5$, $\alpha_1 = 0.03$, $\alpha_2 = 0.06$, $\alpha_3 = 0.07$, $\alpha_4 = 0.08$, $\beta_1 = 0.05$, $\beta_2 = 0.08$, $\beta_3 = 0.12$, $\beta_4 = 0.11$, and $\theta = 0.93$.

Based on Eq. (11), the collective consensus level is obtained, which is $CL(R^*, \ldots, R^*) = 0.8587$. Since $CL(R^*, \ldots, R^*) = 0.8587 < \theta = 0.93$, the consensus-reaching process is utilized to assist the four FMEA experts achieve a consensus.

First, the MCACM is constructed to judge whether the consensus can be reached. Solving the constructed MCACM yields: $\xi_1 = 0.0276$, $\xi_2 = 0$, $\xi_3 = 0$ and $\xi_4 = 0$. Since $\xi_i + \xi_1 + \xi_2 + \xi_3 = 0.0276 > 0$, the FMEA experts need to adjust their compromise values. We suppose that the adjusted compromise values: $\beta_1 = 0.0776$, $\beta_2 = 0.08$, $\beta_3 = 0.12$ and $\beta_4 = 0.11$. Based on these adjusted compromise values, the MCCM with limited compromise and tolerance behaviors are designed. Solving this model, the adjusted LDARMs ($\bar{R}, \bar{R}^2$, $\bar{R}^3$, $\bar{R}^4$) are produced, shown in Tables 9-12.

The adjusted collective LDARM $\bar{R}$ is obtained:

<table>
<thead>
<tr>
<th>FMs</th>
<th>$R_F$</th>
<th>$R_F$</th>
<th>$R_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{M1}$</td>
<td>$(\ell_i, 0.3598), (\ell_i, 0.3402), (\ell_i, 0.3402)$, $T$</td>
<td>$(\ell_i, 0.69), (\ell_i, 0.26), (\ell_i, 0.59)$</td>
<td>$(\ell_i, 0.8422), (\ell_i, 0.0978), (\ell_i, 0.0978)$</td>
</tr>
<tr>
<td>$F_{M2}$</td>
<td>$(\ell_i, 0.58), (\ell_i, 0.3), (\ell_i, 0.12), (\ell_i, 0.12)$</td>
<td>$(\ell_i, 0.05), (\ell_i, 0.1756), (\ell_i, 0.1756)$</td>
<td>$(\ell_i, 0.15), (\ell_i, 0.64)$, $(\ell_i, 0.211)$</td>
</tr>
<tr>
<td>$F_{M3}$</td>
<td>$(\ell_i, 0.7598), (\ell_i, 0.2402), (\ell_i, 0.2402)$</td>
<td>$(\ell_i, 0.31), (\ell_i, 0.69)$, $(\ell_i, 0.1756)$</td>
<td>$(\ell_i, 0.141)$, $(\ell_i, 0.5782), (\ell_i, 0.4218)$</td>
</tr>
<tr>
<td>$F_{M4}$</td>
<td>$(\ell_i, 0.24), (\ell_i, 0.76)$, $(\ell_i, 0.76)$</td>
<td>$(\ell_i, 0.31), (\ell_i, 0.69)$, $(\ell_i, 0.1756)$</td>
<td>$(\ell_i, 0.051), (\ell_i, 0.051), (\ell_i, 0.051)$</td>
</tr>
<tr>
<td>$F_{M5}$</td>
<td>$(\ell_i, 0.1768), (\ell_i, 0.8322), (\ell_i, 0.8322)$</td>
<td>$(\ell_i, 0.057), (\ell_i, 0.7698), (\ell_i, 0.7698)$</td>
<td>$(\ell_i, 0.05), (\ell_i, 0.05), (\ell_i, 0.051)$</td>
</tr>
<tr>
<td>$F_{M6}$</td>
<td>$(\ell_i, 0.5902), (\ell_i, 0.4096), (\ell_i, 0.4096)$</td>
<td>$(\ell_i, 0.1098), (\ell_i, 0.8902), (\ell_i, 0.8902)$</td>
<td>$(\ell_i, 0.0898), (\ell_i, 0.9102)$</td>
</tr>
</tbody>
</table>

The overall evaluation values about the six FMs are yielded using Eq. (8) from LDARM $\bar{R}$:

$EV_i = \{(\ell_i, 0.0403), (\ell_i, 0.5637), (\ell_i, 0.5355), (\ell_i, 0.0425)\}$

$EV_i = \{(\ell_i, 0.0112), (\ell_i, 0.1391), (\ell_i, 0.5626), (\ell_i, 0.2133), (\ell_i, 0.0738)\}$

$EV_i = \{(\ell_i, 0.1378), (\ell_i, 0.3191), (\ell_i, 0.449), (\ell_i, 0.0634), (\ell_i, 0.0308)\}$

$EV_i = \{(\ell_i, 0.0757), (\ell_i, 0.4176), (\ell_i, 0.4434), (\ell_i, 0.0583), (\ell_i, 0.005)\}$
Thus, the risk order of the six FMs is 
\[ FM_5 > FM_3 > FM_2 > FM_1 > FM_6 \]
which indicates that \( FM_5 \) has the highest risk and it should be given more attention for risk elimination.

V. SENSITIVITY AND COMPARATIVE ANALYSIS

This section uses sensitivity and comparative analyses to discuss the validity of the MCC-FMEA framework with limited compromise and tolerance behaviors.

1) Sensitivity analysis

Here, the total consensus cost based on the data used in Section IV, under different parameter combinations, is analyzed. Specifically, the following parameter combinations are used:

\[
\begin{align*}
(S1) & \quad c = (0.4, 0.5, 0.6, 0.7) , \quad \lambda = (0.2, 0.4, 0.3, 0.1)^T , \quad \alpha = 0.03 , \quad \beta = [0.08, 0.09, \ldots, 0.17] , \quad \theta = [0.88, 0.885, \ldots, 0.92] ; \\
(S2) & \quad c = (0.8, 0.6, 0.4, 0.5) , \quad \lambda = (0.15, 0.2, 0.25, 0.4)^T , \quad \alpha = 0.05 , \quad \\
& \quad \beta = [0.08, 0.09, \ldots, 0.17] , \quad \theta = [0.88, 0.885, \ldots, 0.92] ; \\
(S3) & \quad c = (0.7, 0.8, 0.5, 0.6) , \quad \lambda = (0.35, 0.3, 0.2, 0.15)^T , \quad \beta = 0.1 , \quad \\
& \quad \alpha = [0.01, 0.02, \ldots, 0.09] , \quad \theta = [0.88, 0.885, \ldots, 0.93] ; \\
(S4) & \quad c = (0.75, 0.85, 0.7, 0.55) , \quad \lambda = (0.3, 0.35, 0.15, 0.2)^T , \quad \beta = 0.15 , \quad \\
& \quad \alpha = [0.01, 0.02, \ldots, 0.12] , \quad \theta = [0.88, 0.885, \ldots, 0.94] .
\end{align*}
\]

Note 1: The sensitivity analysis is focused on discussing the impact of parameters \( \lambda, \alpha \) and \( \beta \) on the consensus cost. To make the analysis results more reliable, we considered different values for \( c \) and \( \lambda \) in the case of parameter combinations (S1-S4). Notably, similar results can be obtained when using other values for \( c \) and \( \lambda \). After a preliminary analysis, we found that the consensus cost is 0 in most cases when \( \alpha > 0.1 \), and the parameter \( \beta \) has a little influence on the consensus cost when it is greater than 0.16. To better show the influence of \( \alpha \) and \( \beta \) on the consensus cost, we considered that \( 0.01 \leq \alpha \leq 0.12 \) and \( 0.08 \leq \beta \leq 0.17 \) in establishing parameter combinations (S1-S4). Concretely, parameter combinations (S1) and (S2) are used to analyze the impact of parameters \( \beta \) and \( \theta \) on the consensus cost, in which \( c, \lambda \) and \( \alpha \) are fixed. The setting of \( \theta \) depends on \( \lambda \) and \( \beta \). On the other hand, the value of \( \lambda \) should be greater than the consensus level obtained using Eq. (11) under parameter \( \lambda \). On the other hand, the value of \( \lambda \) should be less than the maximum consensus levels obtained from model (23) under each \( \beta \) values (that is, \( 0.08, 0.09, \ldots, 0.17 \) ). For example, the consensus level under \( \lambda = (0.2, 0.4, 0.3, 0.1)^T \) is 0.8606, and the maximum consensus levels under different \( \beta \) values (that is, \( 0.08, 0.09, \ldots, 0.17 \) ) are \( (0.9292, 0.9295, \ldots, 0.9738) \) based on model (23). Accordingly, we considered that \( \lambda = [0.88, 0.92] \) in constructing parameter combination (S1). Similarly, the range of \( \theta \) in parameter combination (S2) can be determined. Moreover, the variations of \( \beta \) and \( \theta \) are set as 0.01 and 0.005, respectively, to better reflect their influence on the consensus cost. It should be noted that similar results can be achieved when increasing the variations. Parameter combinations (S3) and (S4) are utilized to analyze the impact of \( \alpha \) and \( \theta \) on the consensus cost. The basic idea of establishing (S3) and (S4) is similar to (S1) and (S2), and we omit their analysis to save space.

The sensitivity analysis results under different parameter combinations (S1-S4) are obtained and pictured in Figs. 3 and 4.
The MCCM, with limited compromise and tolerance behaviors, is presented to address the following issues: (i) FMEA experts may not tolerate modifying their risk assessments without constraints; and (ii) an FMEA expert will accept the risk assessment suggestion without being compensated for any adjusting costs if the deviation is within his/her tolerance limit. Several desirable properties of the MCCM with limited compromise and tolerance behaviors are analyzed.

(2) To analyze the MCCM solution with limited compromise and tolerance behaviors, the MCACM and the maximum consensus model with limited compromise behaviors are presented.

(3) An interactive MCC-FMEA framework with limited compromise and tolerance behaviors is also designed to achieve a high-acceptability collective solution to the FMEA problem.

(4) The validity of the proposed MCC-FMEA framework was demonstrated via a case study of the assessment of COVID-19-related risk in radiation oncology. Moreover, a comparison analysis indicated that our proposal can reduce consensus costs when compared against existing minimum cost consensus approaches.

This study constructs a connection between the consensus-reaching model and FMEA, which can provide decision support to help FMEA experts reach a consensus regarding their risk assessments. This ability will be key for complex reliability management problems, such as the assessment and management of the risk of SARS-CoV-2 infection in an IVF laboratory [12] and the improvement of the planning and operation of maintenance activities [14].

Meanwhile, three interesting future research directions are provided below:

(1) Since the implementation of FMEA can be regarded as a special GDM, it is very promising to expand our proposal based on some latest GDM methods (e.g., Ranking range based method [31], and preference–approval structures based method [11]).

(2) FMEA experts may present non-cooperative behaviors in the consensus-reaching process [47], for example, some experts may modify their risk assessments to a very low extent, or may even be unwilling to adjust their risk assessments at all, which will affect the quality of the reliability management. Thus, the non-cooperative behaviors of FMEA experts in the MCC-FMEA framework represent an interesting and worthwhile avenue of research.

(3) As a new research field of machine learning and decision analysis, preference learning techniques have been developed recently [29]. The application of a data-driven preference learning method to estimate the relevant parameters (e.g., compromise and tolerance thresholds) involved in the MCC-FMEA framework with limited compromise and tolerance behaviors is a promising future area of research.

REFERENCES


distribution context in group decision making, Group Decision and Negotiation 30 (2021) 97-118.


