Fuzzy convolutional deep-learning model to estimate the operational risk capital using multi-source risk events

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Abstract

Operational Risk (OR) is usually caused by losses due to human errors, inadequate or defective internal processes, system failures or external events that affect an organization. According to the Basel II agreement, OR is defined by seven risk events: internal fraud, external fraud, labour relations, clients, damage to fixed assets, technological failures and failures in the execution & administration of processes. However, due to the large amount of qualitative information, the uncertainty and the low frequency at which these risk events are generated in an organization, their modeling is still a technological challenge. This paper takes up this challenge and presents a fuzzy convolutional deep-learning model to estimate, based on the Basel III recommendations, the OR Loss Component (OR-LC) in an organization. The proposed model integrates qualitative information as linguistic random variables, as well as risk events data from different sources using multi-dimensional fuzzy credibility concepts. The results show the stability of the proposed model with respect to the OR-LC estimation from both structural and dimensional point of views, making it an ideal tool for modeling OR from the perspective of: (a) the regulators (Basel Committee on Banking Supervision) by allowing the integration of experts’ criteria into the OR-LC; (b) the insurers by allowing the integration of risk events from different sources; and (c) organizations and financial entities by allowing the a priori evaluation of the OR-LC of new financial products based on technological platforms and electronic channels.

Keywords: Financial modeling, Stochastic modeling, Fuzzy Cognitive Maps, Log-logistic activation function, Financial Scenarios, Project Finance.

1. Introduction

The Basel Committee of Banking Supervision (Basel II - Agreement) defines Operational Risk (OR) as “the risk that can cause losses due to human errors, inadequate or defective internal processes, system failures as a consequence of external events” [4]. According to the business operations of an organization, OR is defined by seven key risk events [21]: internal fraud, external fraud, labour relations, clients, damage to fixed assets, technological failures and failures in the execution & administration of processes. Meanwhile, the Aggregate Loss Distributions (ALD) represents the statistical loss behavior derived from these risk events, where the Operational Value at Risk (OpVar - 99.9% percentile) determines the minimum regulatory capital, also known as Operational Risk Capital (ORC), to face a catastrophic risk event [26].

According to the Basel III agreement, ALD is represented by the Loss Component (LC), which requires a reliable method to determine its amount in agreement with the regulation: “all systems for estimating the LC component should include the use of internal data, relevant external data, analysis of scenarios and factors that reflect the business environment and internal control systems” [1]. Due to the large amount of qualitative information [62], the uncertainty and the low frequency at which these risk events are generated in an organization, the LC modeling represents a technological challenge from the regulations recommended by the Basel committee [17, 23].
The literature is rich in studies addressing the OR modeling problem but a consolidated solution is yet to be found. Among the proposed approaches the below four promising research methodologies are identified:

1. Within the first research methodology, prominence is given to the developing of frameworks for OR management, mainly in financial and insurance organizations to meet the Basel II requirements [25, 44]. A framework based on historical case studies of the Global Analysts Settlement (GAS) scandal was proposed in [35], while a decision-making framework to evaluate the influence of the risk in the culture of organizations was designed in [54]. Regression analysis is also currently being used to analyse the impacts of corporate governance policies on risk management. Indeed, in [42] the relationship between the disclosure of such policies and risk management to support investors is statistically studied. In relation to this, it is worth mentioning a pilot study for commercial banks that strictly relies on the Basel II agreement and corporate governance policies for risk mitigation [57], which integrates good practices for risk management, based on the “eXposure, Occurrence and Impact” (XOI) method, and a series of Bayesian models. It is noticed, though, that the frameworks and models developed for OR management do not allow for the integration of qualitative information from the risk factors to which an organization is exposed.

2. The second research methodology gives more emphasis to the OR modeling, with a focus on the ORC estimation with Machine Learning (ML) and Artificial Intelligence (AI) methods and models [3]. Among the many models available in literature, it is worth mentioning the following relevant examples: (i) Double Correlation Model (DCM) to estimate ORC taking into account the interactions between frequency and severity for different sets of loss events [59]; (ii) Fuzzy Cognitive Map (FCM) and Bayesian Belief Network (BBN) model to define risk mitigation activities and reduce the corresponding operational losses accordingly [2]; (iii) Adaptive Clustering Model to obtain the optimal partitioning of a risk portfolio, identifying the lowest risks for partitions [16]; (iv) Multi-period Model for calculating ORC to overcome operational disruptions over extended periods in financial services and manufacturing industry [30]; (v) Generalisation of the Autoregressive Conditional Heteroscedasticity (GARCH) models in ORC estimation for Taiwan’s banking industry [12]. The comparative analysis reported in [19] points out strengths and weaknesses of common ORC models such as the Basic Indicator Approach (BIA), the Standardized Approach (SA) and the Alternative Standardized Approach (ASA). It is fair to conclude that the wide spectrum of methods and models for the estimation of ORC fail to address the qualitative elements involved in this process because they do not consider experts’ judgments despite being shown to be a key element in mitigating the OR [62].

3. The third research methodology focuses on the ORC estimation by aggregating information from multiple repositories, on risk event records of financial institutions from all over the world [58], via statistical operators [55]. It is worth pointing out that multi-source approaches, i.e. approaches that use different data-sets, are usually more robust and difficult to design [6], although they do not always help describing the LC in a specific organization. To overcome this issue, AI approaches to address uncertainty and detect general patterns of risk have been proposed to derive more robust and reliable estimates. In [49], a fuzzy logic model was developed to aggregate both quantitative and qualitative information, while machine-learning and probabilistic models were used [36, 13]. The inverse adaptive fuzzy inference model processing distributions of risk events generated via the Monte Carlo simulation presented in [48] allows to predict the evolution of risk losses over time and, consequently, a more informed risk management, while the formulation of management matrices for the minimization of such losses was investigated in [47]. Thus, this research methodology has provided with approaches to integrate qualitative information from the structure’s characterization of risk event sources based on fuzzy logic concepts.

4. The fourth research methodology identifies sources together with multiple cyber-threats and cyber-fraud that can impact the ORC estimation process [14]. Examples of these crimes, more frequent since the introduction of e-banking services [5], include the illegal access and interception, system interference and misuse or piracy of devices for the purpose of achieving a financial advantage [18]. To quantify the potential losses in e-banking transactions, following the methodology presented in [20], a Bayesian Network (BN) model for estimating the ORC value in financial firms was designed in [8]; multiple machine-learning techniques such decision trees in [40]; while stacked self-encoders and Boltzman restricted machines are used to detect credit card fraudulent transactions in [40]. Hence, it is evident that AI and, in particular, one of its most promising but challenging approaches [60], machine-learning, are key in modern finance [44]. This was clear already a decade ago, when predictive models were starting to find their use in the field of risk estimation, credit scoring, and bankruptcy forecasting [56]. More sophisticated solutions were then developed to deal with new risk events, such as those related to cyber-fraud. Machine-learning is changing the way the financial industry works [29], with Deep-Learning (DL) currently being the most researched and applied paradigm due to its versatility and prediction
capabilities. Within this field, Convolutional Neural Networks (CNN) are becoming very popular to unveil operational-risk features that could not be detected with conventional approaches [22].

Following the third and fourth methodologies, this research aims at designing a novel model for the OR modeling in organizations by bringing together their best features. DL will be used to generate the model with fuzzy credibility maps (FCM) [7] to handle both qualitative and quantitative information from multiple databases or loss event sources. To adhere to the guidelines of the Basel II and the Basel III agreements [1], both internal (i.e. within the organisation) and external (i.e. made available by other organisations) databases are considered, which in OR terms are are referred to as Observed Loss Events (OLE) and Available databases of Loss Events (ALE), respectively [50]. The resulting Multi-source Fuzzy Convolutional Deep-Learning (MFC-DL) model is structured in modules as follows:

- **Module 1** integrates a Generalized Logistic Neural Model (GLNM) [24] that uses an auto-encoder strategy [53] for the representation of the random variables of frequency and severity that define a database of loss events as Linguistic Random Variables (LRV);

- **Module 2** integrates a Fuzzy Cognitive Map (FCM) obtained as a result of a convolutional process of the fuzzy sets that define the frequency and severity random variables for each employed database [49];

- **Module 3** integrates an Extended Fuzzy Credibility Map (EFCM) based on credibility theory concepts [9]. This module is constructed through a convolutional process [51] between the two FCMs (OLE-FCM,ALE-FCM) obtained in Module 2;

- **Module 4** leads to the production of a Fuzzy Compact Credibility Map (FCCM) through a random sampling pooling process as shown in [32];

- Eventually, the final **Module 5** integrates a Fully Connected Layer (FCL) [37] inspired by an Adaptive Neuro-Fuzzy Inference System (ANFIS), which is used to estimate the ORC value [50].

The remainder of the article is structured as follows. Section 2 presents the main concepts that support the model. Section 3 details the methodology for the analysis and validation of the proposed model. Section 4 presents the analysis and discussion of the obtained results, according to a series of parameters and metrics that define the general methodology to estimate the. Finally, Section 5 draws the conclusions on this study and proposes future work on forecasting and management of OR in real time.

## 2. Theory

### 2.1. Operational Risk

As mentioned in the introduction, the Basel II agreement defines Operational Risk (OR) as: “the possibility of incurring in losses due to deficiencies, failures or inadequacies, in human resources, processes, technologies, infrastructure or by the occurrence of external events” [48]. The Basel II agreement also establishes the guidelines to estimate the losses due to this risk; among them, we highlight the Basic Indicator Approach (BIA), the Standard Indicator Approach (SIA) and the Advanced Measurement Approaches (AMA) [38, 39].

### 2.2. Aggregate Loss Distribution (ALD)

According to the Advanced Measurement Approach (AMA) introduced in the Basel II agreement [38, 39], the characterization of risks associated to the business operations of an organization (in the last three years period) [1] is done with the ALD (LC-Component) [55]:

\[
ALD(k) = \sum_{i=1}^{N_k} X_i, \quad k \in \mathbb{N}
\]  

(1)

where:

- the severity random variables \((X_i)\) are independent and identically distributed (iid), commonly with a continuous distribution such as the Lognormal, Weibull or Generalized Pareto [39]. These variables describe the magnitude or severity of a loss event;

- the frequency of occurrence of a loss event \(N_k\) is either obtained by using discrete Poisson or Binomial distributions as shown in [45, 39].
According to this approach, every loss event is fully characterized by the frequency \( N_k \) and severity \( X_i \) values. More details on implementing ALD are available in [39, 48]. To put ALD in practice, the ALD distribution as depicted in Fig. 1, which must be evaluated via the Monte Carlo simulation or the Panjer Recursion of Fast Fourier Transform based algorithms [15], is used as a structure to classify losses on three main categories: Expected Losses (EL), with upper limit defined by the ALD mean; the Stress Losses (SL) with lower limit defined by the Operational Value at Risk (OpVar) representing the 99.9% percentile of the ALD distribution and that is named by the Basel III agreement as the Operational Risk Capital (ORC), i.e. the minimum regulatory capital to face a catastrophic loss event; the Unexpected Losses (UL) are located between EL and SL. According to the Basel III agreement, the ALD is defined by the loss component (LC), which extends the internal losses in an organization in the last 10 years period, where the \( y \) axis (density) represents the probability of occurrence of a risk event and the \( x \) axis is the aggregate losses (kUSD) (see Fig. 1) [1]. Since frequency and the severity are independent, it is worth noticing that highly frequent events do not necessarily show a high (or low) severity value. However, events in the EL areas normally show high frequency and low severity while those in the UL area present relatively low frequency and high severity. Finally, events in the SL category, are characterized by low frequency and very high severity, carry losses greater than those in the UL category.

![Figure 1: Loss Distribution Approach (LDA).](image)

2.3. Buhlmann Credibility

Given the observations in previous periods, \( X_1, X_2, X_3, \ldots, X_n \) (for OLE and ALE), and their mean value \( \bar{X} \), the Buhlmann Credibility \( Z \) is defined as:

\[
ORC = Z \cdot \bar{X} + (1 - Z) \cdot \mu
\]

where the expected value of hypothetical mean \( \mu \), commonly referred to as the “unconditional” mean value, is obtained via:

\[
\mu = E[X] = E[E[X | \Theta]] \quad \text{with} \quad \Theta \in \{OLE, ALE\}
\]

while the “Buhlmann credibility” factor \( Z \) is calculated as:

\[
Z = \frac{n}{n + \frac{EPV}{VHM}}
\]

with \( EPV = E[Var[X | \Theta]] \) (Expected Process Variance) and \( VHM = Var[E[X | \Theta]] \) (Variance of the Hypothetical Mean). In the scientific literature, the value \((1 - Z)\) is known as the complementary credibility [39, 49].

2.4. Definitions and Concepts

Some concepts required for the understanding of the remainder of this article are provided below.
**Definition 1** (Linguistic Random Variable (LRV)). A Linguistic Random Variable (LRV) is a random variable expressed according to five quantiles {q₀, q₁, q₂, q₃, q₄}, which define its Cumulative Distribution Function (CDF), representing five fuzzy linguistic labels with Gaussian membership functions for clustering data (Figures 2 (a), (b), (c)).

According to Figure 2, the structure and shape of the fuzzy sets representing a LRV (CDF), where $F$ depend on the skewness index, $a = \frac{\partial F(q_2)}{\partial x}$, as follows [49]:

- if $a < 0$ the CDF is negatively skewed and the data come from a distribution with an *inverted long tail* (unbalanced fuzzy sets with a tendency to the left side – Figure 2 (a)).
- if $a = 1$ the CDF is not skewed and the data come from a symmetrical distribution (balanced fuzzy sets – Figure 2 (b));
- if $a > 0$ the CDF is positively skewed and the data come from a distribution with a *long tail* (unbalanced fuzzy sets with a tendency to the right side – Figure 2 (c));

![Figure 2](image)

**Figure 2**: Different structure for LRVs – (a) Unbalanced fuzzy sets (*skewness* < 0); (b) Balanced fuzzy sets (*skewness* = 0); (c) Unbalance fuzzy sets (*skewness* > 0)

**Definition 2** (*Structural Stability*). A machine-learning model is structurally stable if the data given by the model has the same probability distribution that the reference data used for learning, regardless of the sampling method used to select the input data. This stability can be evidence thought the pivot point that defines the CDF for reference data as shown in Figure 3 (a), (b).

The Basel II agreement suggests the use of long-tail or slender distributions of type Lognormal, Log-logistic, Logistic, Weibull or Generalized Pareto for modeling ALD [39, 27]. Figure 3 shows the probability distribution for different structural parameters according to the CDF defined by Equation (7).

![Figure 3](image)

**Figure 3**: Structural Stability - (a) Cumulative distribution function (pivot point); (b) Probability distribution function
Definition 3 (Dimensional Stability). A machine learning model is dimensional stable if the output data mean \( \mu \) is sensitive to the input data magnitude, keeping the same probability distribution (structural stability) (Figure 4 (a), (b)).

According to Equation (7), dimensional stability is defined by the translation factor \( a \), which represents the sensitivity of the output data, regardless of the sampling method used to select the input data. This behavior also affects the magnitude of the mean for the output data \( \mu \).

\[
\text{ML}_k = \frac{\text{ALE}_{\text{OpVar}}}{\text{OLE}_{\text{OpVar}}} 
\]

where \( k \) is the \( k \)-scenario identifier; \( \text{ALE}_{\text{OpVar}} \) is the OpVar value for a \( k \)-ALE scenario, estimated with a confidence level of 99.9%; \( \text{OLE}_{\text{OpVar}} \) is the OpVar value for the OLE scenario, estimated with a confidence level of 99.9%.

In the context of the theory of credibility, the number of fuzzy sets required for the construction of a FCM for the functional integration of loss events databases is known as the Index of Granularity (IG) [34, 49]:

\[
IG = nfs_{\text{OLE}} \cdot nfs_{l-ALE} 
\]

where \( nfs_{\text{OLE}} \) is the number of fuzzy sets used for modeling ALD modeling for an observed or reference loss events database (Observed loss events - OLE); \( nfs_{l-ALE} \) is the number of fuzzy sets used for the ALD modeling for an available loss events database (Available loss events - ALD); and \( l \) represents the magnitude of losses with regard the magnitude of losses grouped in the OLE database.

3. Methodology

Due to the low frequency and uncertainty risk events are generated with, companies and organizations have to use of external databases to estimate the OpVar value. These external databases contain loss events from different sectors of the economy, however the companies and organizations dismiss aspects such as the magnitude, dispersion and statistical structure of the losses events [49]. For this fact, this section defines some key concepts, introduced in the Basel II and Basel III agreements for the implementation of AMA models to estimate OpVar, that help understand better the computation process of the OpVar value integrating different sources of loss events (\( l-ALEs \)).

3.1. Experimental set-up

A total of 701 records of daily non-materialized cyber-fraud events (in the retail business line of bank) was prepared to validate the proposed model. The OLE records were obtained during the period 2009-2011 (OLE) in a SME (Small and Medium Enterprise) financial enterprise and are measured in thousands of dollars (kUSD). Moreover, 19 ALE databases or scenarios of cyber-fraud events were generated with the Monte Carlo sampling
over the available OLE with the parameter setting shown in Table 1, where ML is obtained as the ratio between the OpVar value for OLE and the OpVar value recorded in ALE as per Definition 4. The l-ALE databases can be considered as different sources of risk events including technological failures and cyber-fraud in electronic transactions in different business lines that were generated inside and outside of a financial entity or organization. It is important to remark that, in accordance with the Basel II and III agreement [1], the transactions in different business lines that were generated inside and outside of a financial entity or organization.

Table 1: OLE and ALE Scenarios – ML, Mean and Variance values are expressed in kUSD; ND refers to the Number of Data-sets or risk scenarios.

<table>
<thead>
<tr>
<th>ALE</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>0.015842</td>
<td>0.039982</td>
<td>0.125266</td>
<td>0.133339</td>
<td>0.13955</td>
<td>0.45507</td>
<td>1.43311</td>
<td>1.516526</td>
<td>1.431425</td>
<td>7.227431</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0092328</td>
<td>0.0180564</td>
<td>0.0730061</td>
<td>0.077743</td>
<td>0.0813299</td>
<td>0.264012</td>
<td>0.845217</td>
<td>0.885381</td>
<td>2.407794</td>
<td>4.212146</td>
</tr>
<tr>
<td>Variance</td>
<td>0.000156</td>
<td>0.0007396</td>
<td>0.0112171</td>
<td>0.0075936</td>
<td>0.0131748</td>
<td>0.144555</td>
<td>1.543626</td>
<td>1.546756</td>
<td>11.955969</td>
<td>35.607113</td>
</tr>
<tr>
<td>ND</td>
<td>2763</td>
<td>4986</td>
<td>5340</td>
<td>67</td>
<td>5308</td>
<td>2871</td>
<td>4818</td>
<td>5448</td>
<td>6250</td>
<td>4228</td>
</tr>
<tr>
<td>OpVar</td>
<td>0.1078375</td>
<td>0.234677</td>
<td>0.546098</td>
<td>0.0180564</td>
<td>0.0730051</td>
<td>0.0112171</td>
<td>0.0075936</td>
<td>0.0131748</td>
<td>0.144555</td>
<td>1.543626</td>
</tr>
</tbody>
</table>

3.2. Multi-source Fuzzy Convolutional Deep Learning Model (MFC-DL)

The five modules forming the proposed model are thoroughly described in this section. As a whole system, it can be seen as a deep-learning convolutional neural model empowered with fuzzy cognitive maps [41].

3.2.1. Generalized Logistic Neural Model (LGNM) (module 1)

The first module allows the representation of frequency and severity random variables as linguistic random variables (LRVs) using a neural model structure, which was set-up using and auto-encoder strategy [10]. This model integrates into its output an activation function based on a Generalized CDF (GCDF), configuring a Generalized Logistic Neural Model (GLNM) [53, 61], whose output $y_{r,i,k}(s_{i,j,k})$ is mathematically expressed as:

$$y_{r,i,k} = \frac{\beta}{\alpha} \cdot \left( \frac{2 \cdot ys_{i,j,k} + a}{\alpha} \right)^{\beta-1} \left( 1 + \left( \frac{2 \cdot ys_{i,j,k} + a}{\alpha} \right)^{\beta} \right)^{-2}$$

where $i = 0, 1, 2, \ldots, ni$ indicates the $i^{th}$ input LRV data ($ni = number of inputs$); $j = 0, 1, 2, \ldots, no$ indicated the $j^{th}$ hidden neuron ($no = number of hidden neurons$); $k = 0, 1, 2, \ldots, nd$ indicates the records that make up the data set; $\alpha$ is a shape factor ($\alpha > 0$); $\beta$ is the scale factor ($\beta > 0$); $a$ is the translation factor; $ys_{i,j,k}$ is the adaptive linear combiner, commonly formulated by means of a point product of two vectors, $c_{i,j,k}$ and $h_{j,k}$, as follows:

$$ys_{i,j,k} = c_{i,j,k} \cdot h_{j,k}$$
where \( x_{i,k} \) is a generic (input) vector for frequency (i.e. \( x_{1,k} \)) or severity (i.e. \( x_{2,k} \)) random variables; \( w_{j,i,k} \) represents the connection between the \( i^{th} \) input variable and the \( j^{th} \) hidden neuron for the \( k \) record; \( c_{i,j,k} \) represents the connection between the \( j^{th} \) hidden neuron and the \( i^{th} \) input-output variable for the \( k \) record; and \( e_{j,k}^2 \) is the Mean Square Error (MSE) (12). The auto-encoder strategy is supported by the Generalized delta rule (GDR) ((11)-(12)) for connection weights \( c_{i,j,k} \) and \( w_{j,i,k} \) [28].

The fuzzy sets for frequency and severity are obtained based on Definition 1. The fuzzy sets are expressed as: \{\( XC_{q_1,i}, XC_{q_2,i}, XC_{q_3,i} \), \( XC_{q_4,i} \}\}, where the size of their base will be determined by the dispersion of the data grouped by the already mentioned fuzzy sets: \{\( \sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4 \}\}. In general, each fuzzy set can be expressed as follows:

\[
FLS_{j,i} = e^{-\frac{1}{2} \left( \frac{XC_{j,i} - x_{i,k}}{\sigma_{j,i}} \right)^2}
\]

where \( FLS_{j,i} \) (\( j = 0, 1, 2, 3, 4 \)) are the membership functions of the fuzzy sets for the LRV \( i = f(\text{frequency}), s(\text{severity}) \); and \( k \) is the number of loss events for an \( i-RV \).

### 3.2.2. Fuzzy Convolutional Map (module 2)

The second module integrates a Fuzzy Cognitive Map (FCM) obtained as result of a convolutional process between the fuzzy sets that make up the LRV for frequency and severity for a specific \( l \)-ALE or OLE database. The FCM process is defined as follows:

\[
FCM_{l_{j_1,j_2}} = e^{-\frac{1}{2} \left( \frac{XC_{j_1,f} - x_{f,k}}{\sigma_{j_1,f}} \right)^2} \cdot e^{-\frac{1}{2} \left( \frac{XC_{j_2,s} - x_{s,k}}{\sigma_{j_2,s}} \right)^2}
\]

where \( XC_{j_1,f}, XC_{j_2,s} \) are the centroids for the LRVs random variables of frequency \((j_1,f)\) and severity \((j_2,s)\), respectively; \( \sigma_{j_1,f}, \sigma_{j_2,s} \) are the standard deviations for each fuzzy set that make up the LRVs of frequency \((j_1,f)\) and severity \((j_2,s)\), respectively.

The FCM shows the dis-aggregated distribution of losses according to each of fuzzy sets that make up the LRVs for frequency and severity, and for a specific database of loss events \((l \text{ - ALE})\). Here, in general, a FCM will have a total of 25 Gaussian bells membership functions \((IG = 25)\).

### 3.2.3. Extended Fuzzy Credibility Map (module 3)

As a result of the convolutional process between two FCMs representing two databases of loss events (OLE - \( l \text{ - ALE} \) databases), the third module is designed to create an Extended Fuzzy Credibility Map (EFCM) based on the Buhlmann theory [55]. According to the Buhlmann theory, the Variance of the Hypothetical Mean (VHM) for a random variable \( v \) of frequency or severity \((v \in \{ f, s \})\), for \( l_1 \) of OLE databases of loss events and \( l_2 \) ALE databases of loss events can be expressed as follows:

\[
VHM_{v,j_1,j_2} = nd_{l_1} \cdot (XC_{l_1,j_1})^2 + nd_{l_2} \cdot (XC_{l_2,j_2})^2 - \mu_{v,(l_1,j_1),(l_2,j_2)}^2
\]

\[
\mu_{v,(l_1,j_1),(l_2,j_2)} = \frac{nd_{l_1} \cdot XC_{l_1,j_1} + nd_{l_2} \cdot XC_{l_2,j_2}}{nd_{l_1} + nd_{l_2}}
\]

where \( nd_{l_1}, nd_{l_2} \) refer to the number of risk events in database \( l_1 \) and \( l_2 \), respectively; and \( \mu_{v,(l_1,j_1),(l_2,j_2)} \) is the hypothetical mean for the random variable \( v \), databases \( l_1, l_2 \) and clusters \( j_1, j_2 \).

The Expected Process Variance (EPV) for the random variable \( v \), databases \( l_1, l_2 \) and clusters \( j_1, j_2 \) is expressed as

\[
EPV_{v,j_1,j_2} = nd_{l_1} \cdot \sigma_{j_1}^2 + nd_{l_2} \cdot \sigma_{j_2}^2
\]

The Credibility factor \((kc)\) for the random variable \( v \), databases \( l_1, l_2 \), cluster \( j_1, j_2 \) is defined as [49]

\[
kc_{v,j_1,j_2} = \frac{EPV_{v,j_1,j_2}}{VHM_{v,j_1,j_2}}
\]

The Extended Fuzzy Credibility Map (EFCM) is expressed as follows [49]:

\[
EFCM_{m_1,m_2} = e^{-\frac{1}{FS} \left( EPV_{v,j_1,j_2} \right)} \cdot e^{-\frac{1}{FS} \left( EPV_{v,j_1,j_2} \right)}
\]

where \( FS \) is the number of loss events for an \( i-RV \).
where \( m_{1,f} = j_1 + j_2, \) i.e. \( (m_{1,f} = 1, 2, 3, \ldots, 25); \) \( m_{2,s} = j_1 + j_2, \) i.e. \( (m_{2,s} = 1, 2, 3, \ldots, 25); \) and \( FS \) is the Scale Factor used to control the effect of the standard deviation of fuzzy sets on the credibility estimation.

The EFCM establishes a framework to understand the qualitative-quantitative structure of the credibility after the integration of two FCMs, according to the self structure of the fuzzy sets that make up their structure. According to the structure of fuzzy sets that make up the LRVs for each database, this EFCM will have a total of 625 Gaussian bells (\( IG=625 \)). It is also important to highlight that the FS allows to align the credibility estimated as a result of a convolutional process between two FCMs. This alignment determines the maximum value for the Z-credibility (diagonal dominance - DD) for EFCM structure. This process is described in [49].

### 3.2.4. Sampling Layer – Fuzzy Compact Credibility Maps (FCCMs) (module 4)

The fourth module integrates a sampling mechanism for creating a Fuzzy Compact Credibility Map (FCCM) from an EFCM structure [32], which is expressed as follows:

\[
Zms_{k_1,k_2} \sim EFCM (\mu_{k_1,k_2}, \sigma_{k_1,k_2})
\]

where \( \mu_{k_1,k_2} \) and \( \sigma_{k_1,k_2} \) represent the mean and standard deviation for the credibility values grouped in the window \( k_1,k_2 \). In general, the window structure is defined for a size of \([k_1 − 12 : k_1 + 12;k_2 − 12 : k_2 + 12]; \) \( k_1 = 13, 38, 63, \ldots, 613 \) \((nk_1 = 25)\); \( k_2 = 13, 38, 63, \ldots, 613 \) \((nk_2 = 25)\).

### 3.2.5. Fully Connected Layer (module 5)

The Fully Connected Layer (FCL) allows the estimation of \( OpVar \) values as follows [48, 49]:

\[
y^{\text{OpVar},k} = \sum_{j_5=1}^{n_5} c_{j_6} \left( \sum_{j_4=1}^{n_4} w_{j_5,j_4} \cdot h_{j_5,j_4,k} \right)
\]

\[
h_{j_5,j_4,k} = Zms_{j_5,j_4} \cdot \text{FCM}_{j_5,j_4,OLE} + (1 - Zms_{j_5,j_4}) \cdot \text{FCM}_{j_6,\text{OLE},k,j_5,j_4}
\]

where \( n_5 \) is the number of scenarios or \( l - ALE \) database of loss events; \( k \) is the number of loss events at a confidence level of 99.9\% \((k = 1000); \) \( y^{\text{OpVar},k} \) is the \( OpVar \) for the \( k \) sampling value (loss event) obtained for an \( OLE \) database with regard to \( j_6 - ALE \) database or risk scenario; and \( h_{j_5,j_4,k} \) describes the membership value for the \( j_5,j_4 \) Gaussian bell component for \( Zms \) (Credibility value) for a \( k \) sampling value for \( OLE \) and \( l - ALE \) databases. In general, the \( 1 - Zms \) value is known as the external credibility.

### 3.3. Case study

Based on the credibility modules forming the proposed model, the process to obtain an FCM for the OLE severity versus an \( l - ALE \) external is described below.

1. According to the convolutional module (Module 1) and according to the \( k\)-means clustering process, the frequency and severity random variables forming the OLE database are described in Table 2.

<table>
<thead>
<tr>
<th>Frequency LRV - OLE</th>
<th>( q^0 )</th>
<th>( q^1 )</th>
<th>( q^2 )</th>
<th>( q^3 )</th>
<th>( q^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( XC_{j,f} )</td>
<td>2.00000</td>
<td>4.725000</td>
<td>5.20000</td>
<td>5.75000</td>
<td>8.00000</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>8.23010</td>
<td>4.40711</td>
<td>4.31358</td>
<td>4.52500</td>
<td>7.27142</td>
</tr>
<tr>
<td>ND</td>
<td>2</td>
<td>672</td>
<td>10</td>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Severity LRV - OLE</th>
<th>( q^0 )</th>
<th>( q^1 )</th>
<th>( q^2 )</th>
<th>( q^3 )</th>
<th>( q^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( XC_{j,s} )</td>
<td>1.55078</td>
<td>8.01424</td>
<td>10.91685</td>
<td>14.79250</td>
<td>17.31290</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>23.52205</td>
<td>13.51336</td>
<td>12.33200</td>
<td>15.57757</td>
<td>19.51919</td>
</tr>
<tr>
<td>ND</td>
<td>2</td>
<td>672</td>
<td>10</td>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Fuzzy sets for frequency and severity for the OLE database of loss records

2. According to the central clusters for frequency and severity \((XC_{q_2,f}, XC_{q_2,s})\) for the OLE database, the FCM is expressed as:

\[
\text{FCM}_{q_2,q_2} = e^{-\frac{1}{2} \left( \frac{5.2000 - f_{f,k}}{4.31358} \right)^2} - e^{-\frac{1}{2} \left( \frac{10.91685 - f_{f,k}}{12.33200} \right)^2}
\]
Based on the quantiles that define the OLE and 10 – ALE databases, the central value of fuzzy credibility value are obtained:

<table>
<thead>
<tr>
<th></th>
<th>OLE Database</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>qo</td>
</tr>
<tr>
<td>$XC_{j,f}$</td>
<td>1.42422</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>21.83573</td>
</tr>
<tr>
<td>ND</td>
<td>545</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ALE Database</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>qo</td>
</tr>
<tr>
<td>$XC_{j,s}$</td>
<td>0.37907</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>14.73276</td>
</tr>
<tr>
<td>ND</td>
<td>3914</td>
</tr>
</tbody>
</table>

Table 3: Fuzzy sets for severity for OLE and ALE databases of loss events

According to Eq. (19), the extended fuzzy credibility for the central value that make up the ECFM for theses databases can be expressed:

$$u_{q_2,q_2} = \frac{10.21375 \times 7 + 4.71065 \times 47}{7 + 47} = 5.42402$$ (24)

$$VHM_{q_2,q_2} = 7 \times 10.21375^2 + 47 \times 4.71065^2 - 5.42402^2 : VHM_{q_2,q_2} = 1743.76876$$ (25)

$$EPV_{q_2,q_2} = 7 \times 14.72556^2 + 47 \times 8.74213^2 : EPV_{q_2,q_2} = 5109.86423$$ (26)

The central value for the FCM for OLE and 10 – ALE databases can be explained according to (20):

$$Zms_{13,13} = e^{-\frac{1}{20}(5109.86423)} : Zms = 0.86371 : if \ FS = 20$$ (27)

Taking as reference the OpVar values for OLE (i.e. $OpVar_{OLE_k} = 15.59501082$) and 10 – ALE (i.e. $OpVar_{10–ALE_k} = 9.982319181$) databases, the OpVar value can be expressed in terms of Equation (22):

$$h_{q_2,q_2} = 0.86371 \times 15.59501 + (1 - 0.86371) \times 9.98231$$ (28)
3.4. Credibility metrics

For the analysis and validation of the proposed model against the OpVar estimation based on Multi-source loss events, the fuzzy model proposed by [46] is used. This model integrates seven statistical metrics that measure the performance of an adaptive model in terms of error against the reference data: Fractional Bias (FB), Normalized Median Square Error (NMSE), Geometric Mean bias (MG), Geometric Variance bias (GV), Factor of Two (FAC2), Index of Agreement (IOA), Unpaired Accuracy of Peak (UAPC2) and Mean Relative Error (MRE).

The below five statistical indicators [49] are used to analyse the structural and dimensional stability of the ALD estimated by the proposed model (Figure 1):

- Skewness Index (SI) to statically describe the shape of the ALD distribution;
- Maximum log-likelihood to select the best probability distribution that represents ALD [33];
- OpVar to indicate the limit for the catastrophic losses for a 99.9% percentile of ALD (transferable losses) (kUS$);
- Expected Losses (EL) to indicate the limit of assumable losses for ALD distribution (kUS$) (Figure 1).
- Unexpected losses (UL) to obtain the mean of losses located between the EL and OpVar limits (manageable losses) (kUS$)

The coverage determines the OpVar percentage composition in terms of EL and UL losses. These percentages also determines the coverage of operational losses in an organization or financial entity:

- Coverage of EL (CEL - index) indicates the percentage of coverage of EL for OLE database:
  \[ CEL = 1 - \frac{EL}{LC_{99.9\%}} \] (29)
- Coverage of UL (CUL - index) indicates the percentage of coverage of UL for OLE database:
  \[ CUL = 1 - \frac{UL}{LC_{99.9\%}} \] (30)

3.5. MFC-DL experimental validation

For the analysis and general validation of each of the modules that make up the MFC – DL model, a set of 19 – ALEs scenarios of cyber-fraud using the Monte Carlo sampling process on the reference database OLE were defined (Table 1).

According to Module 1 of the MFC – DL model, the first stage configures the LRVs for each l – ALE (i.e. frequency, severity) using three clustering adaptive models: Multi-adaptive linear model (MADALINE) [28], Radial Basis Function Neural Network (RBFNN) [11] and adaptive k – means clustering [43]. The first two models are independently configured using an auto-encoder learning strategy for the random variables of frequency and severity, while the third one is independently evaluated based on the minimum variance between the clusters for the random variables of frequency and severity. Each LRV is described using five (5) labels: Very Low, Low, Medium, High and Very High, meeting the quantiles that define a GCDF (Definition 1). At this stage, it is expected that the adaptive clustering models exhibit near-zero asymptotic behaviors (\( e_k^2 \) - Equation (8)) for a total of 1000 cycles of learning. According to the Basel II agreement [1] and Definition 1 [49], high SI is expected for the LRV of severity, mainly due to the structure of losses which usually present slender long-tail distributions (lognormal, loglogistic, Weibull or Generalized Pareto as aforementioned). This same agreement recommends the Poison or Binomial distributions for the modeling of the LRV of frequency, so the SI is expected to be close to zero.

The second stage of the experimental validation proceeds with the adjustment and evaluation of the convolutional mechanisms that integrate the MFC – DL (i.e FCM (Module 2), ECFM (Module 3)) against the estimation of the credibility (i.e. Zms). For this evaluation, two databases of loss events comparable in magnitude and variance were selected (OLE, 1-ALE) as reference, as well as four models commonly used for credibility estimation (i.e. Zms), and which were identified in the scientific literature review: Buhlmann-Straub credibility model (Z – BS) [55], Bayesian Belief Network (Z – BNN) [2], Adaptive clustering model (Adaptive k-means - Z – ACM) [16], and Fuzzy cognitive maps (Z – FCM) [10, 49]. Each of the models will produce a complementary credibility curve (i.e. 1 – Zms) as a result of varying the FS factor (19), which will allow the loss variance for the 1 – ALE to be expanded. In a first phase of this stage of validation, models with higher granularity are expected to generate a larger reduction of
the complementary credibility (i.e., $1 - Z_{ms}$) due to the smoothing effect of the FCMs in the coverage of solution spaces with extended losses. This smoothing makes extreme losses move significantly away from the OLE losses of reference used to estimate credibility (i.e., $Z_{ms}$).

In a second phase of this same stage, the convolutional mechanisms ECFMs are adjusted for each of the 19 – ALEs defined in Section 3.1. Here, the main diagonal that defines the structure of the ECFM is expected to reach significantly high credibility values close to one for different $FS$ values, aiming at reaching the diagonal dominance ($DD$) in this structure. It is important to mention that the convolutional stability of the MFC – DL will be determined by the balance point between the $DD$ value and $Z_{ms}$. Finally, the $MFC – DL$ is evaluated against the estimation of $OpVar$ for each $l – ALE$, and selecting two representative models for its relevance in the scientific literature of the four credibility models described above: Bühlmann-Straub credibility model ($Z – BS$), and Fuzzy cognitive maps ($Z – FCM$). Here, the ECFMs that make up the $MFC – DL$ model are expected to present a better risk mitigation, due to better granularity in the modeling of RVs as LRVs ((14)).

The third stage of the experimental validation proceeds with the set up of the FCL (module 5) for a total of 1000 learning cycles ($MFC – DL – LC$). For each learning cycle, $k : 1000$ risk scenarios are defined as a result of a general random sampling on the available 19 – ALE risk scenarios. Each sampling risk scenario $l – ALE_k$ is defined by $ndl_k$ records. For each $l – ALE_k$, the $y_{dOpVar,k}$ value was estimated ((21)) taking as a reference the OLE database ($FCM – LC$).

In first phase of the third stage, the $MFC – DL (FCL)$ is evaluated using the error metrics proposed by [46]. Here, the LC distributions given by the $MFC – DL$ after a learning process are expected to show similar structures to the distributions suggested by the Basel II agreement for modeling this type of risk (i.e., slender distributions - long tail structures). It is also expected that the LC will present much lower losses that the models used as reference to estimate the $OpVar$ (i.e., $BS – CL, FCM – LC$), due to better characterization of credibility according to the structure of EFCM, which presents credibility values differentiated by zones within the same structure.

In second phase of the third stage, the structural and dimensional stability analysis are carried out. Here, the $MFC – DL$ is evaluated for three additional risk scenarios in absence of a learning mechanism. Each risk scenario has a total of $k : 1000 l – ALE_k$ obtained as result of a random sampling on: (i) a first group of risk scenarios with $ML \leq 1.0$ ($FCM – DL(ML \leq 1.0)$); a second group of risk scenarios with $ML > 1.0$ ($FCM – DL(ML > 1.0)$); and a final third group of scenarios ($FCM – DL(ML \neq 1.0)$) to cover all the risk scenarios available for this study. As per Definition 2, structural stability is evaluated based on the LC structures given by the $MFC – DL$ for each risk scenario, according to the LC distributions suggested by the Basel II agreements for modeling this type of risk.

For structural stability, the CEL and CUL must maintain values similar to the risk scenarios established as reference. Meanwhile, as per Definition 3, dimensional stability is evaluated through the loss events magnitude that make up the LC component, which can be greater or lowers according to the magnitude of the losses grouped in each scenario, keeping the same probability distribution. This fact shows the sensitivity of the $MFC – DL$ against the losses magnitude for any $l – ALE_k$ risk scenario.

4. Experimental Results

The results are grouped in three sections to distinguish between those obtained during the design phase, those relative to performed credibility analysis and those for evaluating the performances in estimating the LC or the ALD distribution.

4.1. Design of the Fuzzy Convolutional Layers

Table 4 and Figure 6 show the behavior of the adaptive clustering methods (MADALINE, RBFNN, LGNM (module 1) and adaptive k-means) used to configure the LRVs for frequency (5) and severity (5) for the OLE database.
Table 4: MSE values $e_k^2$ for clustering adaptive models (smallest value boldfaced).

<table>
<thead>
<tr>
<th>k</th>
<th>MADALINE</th>
<th>RBFNN</th>
<th>LGNM</th>
<th>k-means</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>6.0894128</td>
<td>325.27646</td>
<td>24.821077</td>
<td>165.75922</td>
</tr>
<tr>
<td>200</td>
<td>0.0586388</td>
<td>80.659035</td>
<td>29.086145</td>
<td>31.77443</td>
</tr>
<tr>
<td>300</td>
<td>0.0015837</td>
<td>56.29818</td>
<td>17.374706</td>
<td>26.513417</td>
</tr>
<tr>
<td>400</td>
<td>2.6E-15</td>
<td>51.795922</td>
<td>13.644132</td>
<td>31.043713</td>
</tr>
<tr>
<td>500</td>
<td>4.3E-07</td>
<td>50.335411</td>
<td>14.408815</td>
<td>29.224988</td>
</tr>
<tr>
<td>600</td>
<td>7.089E-09</td>
<td>49.44566</td>
<td>13.454926</td>
<td>31.134178</td>
</tr>
<tr>
<td>700</td>
<td>1.168E-10</td>
<td>48.69342</td>
<td>12.76263</td>
<td>28.351621</td>
</tr>
<tr>
<td>800</td>
<td>1.926E-12</td>
<td>47.981228</td>
<td>12.09563</td>
<td>27.959959</td>
</tr>
<tr>
<td>900</td>
<td>3.17418E-14</td>
<td>47.3031807</td>
<td>11.5854931</td>
<td>28.233197</td>
</tr>
<tr>
<td>1000</td>
<td>5.231E-16</td>
<td>46.652324</td>
<td>11.1508846</td>
<td>26.7863144</td>
</tr>
</tbody>
</table>

Table 5: Clustering process for frequency as linguistic random variable (SI=0.52177)

<table>
<thead>
<tr>
<th>Index</th>
<th>Minimum</th>
<th>Very Low</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
<th>Very High</th>
<th>Maximum</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantiles</td>
<td>0</td>
<td>0.1</td>
<td>0.169230</td>
<td>0.307692</td>
<td>0.446153</td>
<td>1.0</td>
<td>1.0</td>
<td>1.15696</td>
</tr>
<tr>
<td>k-means</td>
<td>0</td>
<td>0.00510</td>
<td>0.17530</td>
<td>0.27680</td>
<td>0.44970</td>
<td>0.54950</td>
<td>1.0</td>
<td>0.79386</td>
</tr>
<tr>
<td>MADALINE</td>
<td>0</td>
<td>0.00767</td>
<td>0.165346</td>
<td>0.63576</td>
<td>0.52349</td>
<td>0.68674</td>
<td>1.0</td>
<td>0.78885</td>
</tr>
<tr>
<td>RBFNN</td>
<td>0</td>
<td>0.00419</td>
<td>0.18868</td>
<td>0.28678</td>
<td>0.35222</td>
<td>0.43012</td>
<td>1.0</td>
<td>0.84410</td>
</tr>
<tr>
<td>LGNM</td>
<td>0</td>
<td>0.00819</td>
<td>0.12642</td>
<td>0.19933</td>
<td>0.39934</td>
<td>0.59938</td>
<td>1.0</td>
<td>0.7147</td>
</tr>
</tbody>
</table>

Table 6: Results of clusterization process for Severity as linguistic random variable (SI=2.97240 (kUS$))

<table>
<thead>
<tr>
<th>Index</th>
<th>Minimum</th>
<th>Very Low</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
<th>Very High</th>
<th>Maximum</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantiles</td>
<td>0</td>
<td>0.1</td>
<td>0.12602</td>
<td>0.15584</td>
<td>0.21477</td>
<td>1.0</td>
<td>1.0</td>
<td>1.75153</td>
</tr>
<tr>
<td>k-means</td>
<td>0</td>
<td>0.16250</td>
<td>0.26600</td>
<td>0.31700</td>
<td>0.18850</td>
<td>0.54360</td>
<td>1.0</td>
<td>1.76648</td>
</tr>
<tr>
<td>MADALINE</td>
<td>0</td>
<td>0.13743</td>
<td>0.24391</td>
<td>0.45864</td>
<td>0.49825</td>
<td>0.74912</td>
<td>1.0</td>
<td>1.54779</td>
</tr>
<tr>
<td>RBFNN</td>
<td>0</td>
<td>0.07851</td>
<td>0.14578</td>
<td>0.21427</td>
<td>0.33524</td>
<td>0.46562</td>
<td>1.0</td>
<td>2.00272</td>
</tr>
<tr>
<td>LGNM</td>
<td>0</td>
<td>0.000795</td>
<td>0.04853</td>
<td>0.16217</td>
<td>0.19919</td>
<td>0.20903</td>
<td>1.0</td>
<td>2.08681</td>
</tr>
</tbody>
</table>

Numerically, Table 4 shows different check points from 100 to 1000 iteration cycles. Here, the smallest error was obtained by the MADALINE model (i.e. $e_k^2: 5.2318E-16$), followed by the proposed LGNM model (i.e. $e_k^2: 11.1508846$). In comparison, results obtained with the adaptive $k$-means algorithm (i.e. $e_k^2: 26.7863144$) and the RBF (i.e. $e_k^2: 46.652324$) are not competitive. In the context of this study, the LGNM presented a good behavior in general.

Table 5 and 6 show the clustering models configuration of the fuzzy sets for each LRV for the $OLE$ database, having as a reference the SI. For frequency, the SI values were located above the statistical general SI ($SI = 0.52177$), and LGNM and MADALINE confirmed to be the best two algorithms. For severity, the SI values were located slightly below the statistical general SI ($SI = 2.97240$), and RBFNN and LGNM were the algorithms with closest SI values. These positive SI values clearly indicate the presence of slender distributions with long tails, predominately for the LRV of severity [46]. Hence, even though MADALINE shows extremely good performances in terms of MSE, the SI values indicating the presence of slender distribution as suggest the Basel II agreement for the modeling of the severity, makes the LGNM to be the most versatile and promising method in the context of the problem.

Figure 7 shows the structure and shape of the fuzzy sets that make up the LRVs for the frequency and severity. According to Definition 1, it is evident that the fuzzy sets are both asymmetric and unbalanced towards the left (i.e. lower losses) for both LRVs. This fact reinforces the presence of long-tail distributions that are characteristic of this type of risk.

4.2. Coverage Analysis for Convolutional Mechanisms

Figure 8 shows the complementary credibility curves obtained by the models $Z-BNN$, $Z-BS$, $Z-ACM$, $Z-FCM$ and $Z-MFC-DE$ in the credibility estimation. Here, models $Z-BNN(1-Zms = 0.531095)$ and $Z-BS(1-Zms = 0.513657)$ had the highest complementary credibility values, as the $FS$ factor was widening the variance of losses for $1-ALE$. This behaviour was mainly due to the low granularity of these models in modeling.
the FMCs used to characterize of losses (\(IG = 1 : (1\text{-distribution}) OLE \cdot (1\text{-distribution}) l − ALE\)), favoring the presence of extreme losses, which has a significant impact on the magnitude of \(OpVar\).

On the other hand, models \(Z − ACM\) (\(1 − Zms = 0.324652\)) and \(Z − FCM\) (\(1 − Zms = 0.197898\)) were configured taking as reference a total of 5-fuzzy sets for modeling the \(ALD\) that defines each of the reference databases (\(OLE, 1 − ALE\)). The credibility curves (i.e. \(1 − Zms\)) showed a greater reduction of complementary credibility, indicating more flexible \(FCMs\) due to a higher granularity (\(IG = 25 : (5 \text{ fuzzy sets}) ALD − OLE \cdot (5 \text{ fuzzy sets}) ALD − 1 − ALE\)). A better characterization of losses in the estimation of credibility [49] brings a positive impact on the reduction of the \(OpVar\) value. It is important to mention the greater flexibility in the representation of losses shifts away the extreme losses from the losses grouped in the \(OLE\) database.

For its part, the \(MFC − DL\) model reached a greater reduction in the estimation of complementary credibility (\(1 − Zms = 0.011109\)), mainly promoted by a higher granularity derived from the convolutional process that a \(EFCM\) structure integrates. A first convolutional process (Module 2) allows the representation of \(FCMs\) (\(IG = 25 : (5 \text{ fuzzy sets})\) frequency \(\cdot (5 \text{ fuzzy sets})\) severity) by decomposition of each database (\(OLE, 1 − ALE\)) using the linguistic random variable for frequency and severity [47, 49], and a second convolutional process (Module 3) for credibility estimation that results in a \(EFCM\) by integration of \(FCMs\) representing the reference databases in this state (\(IG = 625 : (25 \text{ Gaussian bells}) OLE \cdot (25 \text{-Gaussian bells}) 1 − ALE\)). The presence of more flexible \(FCMs\) generated and accelerated smoothing by overlapping of the \(Gaussian\) \(bells\) that represent the losses as the \(FS\) increased. This fact led to a further reduction than in the previous case, which will have an even more significant impact on the reduction of the \(OpVar\) value.

Table 7 shows the adjustment of the convolutional mechanism that allows obtaining \(EFCMs\) by integration of \(FCMs\), taking as reference the databases (i.e. \(OLE, 1 − ALE\)) and three parameters: \(FS\) (Scale Factor), \(DD\) (Diagonal Dominance) defined in [51] and the Convolutional Credibility (CC or \(Zms\)). Table 7 also shows how the increase of the \(FS\) value leads to an increase of \(DD\) to its maximum value when the \(FS\) factor is around 20, after which the \(DD\) decreases. Regarding the CC (\(Zms\)) index, this in general decrease when the \(FS\) increases. It is important to highlight that the most promising values for \(FS\) are those with CC values above 90\% on average. It has to be noticed that the equilibrium point for \(FS\) is located close to 10, and establishes the convolutional value of reference (pivot point) for the integration of databases with loss events of different magnitude. This process was carried out for each \(l − ALE\) available for this study, and it was also widely described in [51].

Figure 9 illustrates the structure of the \(EFCM\), which is obtained through a convolutional process between the databases of reference with \(FS = 10\). Here, the surface and their top view show the \(DD\) of the main diagonal according to \(Zms\) values (\(Zms \approx 1.0\)), which guarantees the stability of the convolutional mechanism defined by Module 4. We can highlight in this same Figure the symmetry of the \(EFCM\) with regard to the main diagonal for an \(Fs\) value close to the aforementioned equilibrium value (\(FS = 10\)). Meanwhile, Figure 10 shows the evolution of the main diagonal of the \(EFCM\) for three different values of \(FS\) (\(FS = 5, FS = 10, FS = 15\)), which shows the sensitivity of the proposed \(MFC − DL\) model to the integration of databases with different magnitude after the pivot point was adjustment. This convolutional stability analysis was replicated for all \(l − ALE\) databases for an initial model set-up, following the methodology proposed by Peña et al. [51].

### 4.3. Credibility analysis using multiple \(ALE\) databases

Table 9 shows the behavior of \(MFC − DL\) with respect to both the credibility (\(Zms\)) and the \(OpVar−OpV(kU\$)\) estimation (i.e. \(MFC − DL − Zms\) Credibility, \(MFC − DL − OPV\) Operational value at risk) for each \(l − ALE\)
<table>
<thead>
<tr>
<th>k</th>
<th>FS</th>
<th>Dominance (DD)</th>
<th>Credibility (CC)</th>
<th>k</th>
<th>FS</th>
<th>Dominance (DD)</th>
<th>Credibility (CC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.012327</td>
<td>0.996772</td>
<td>10</td>
<td>6</td>
<td>0.432355</td>
<td>0.887672</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.070057</td>
<td>0.985924</td>
<td>11</td>
<td>7</td>
<td>0.465322</td>
<td>0.832748</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.316778</td>
<td>0.924820</td>
<td>12</td>
<td>8</td>
<td>0.620118</td>
<td>0.775024</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.862035</td>
<td>0.751828</td>
<td>13</td>
<td>9</td>
<td>0.823548</td>
<td>0.764924</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>1.929394</td>
<td>0.428688</td>
<td>14</td>
<td>10</td>
<td>1.130032</td>
<td>0.528056</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>1.615145</td>
<td>0.133532</td>
<td>15</td>
<td>11</td>
<td>1.276008</td>
<td>0.671764</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>0.838837</td>
<td>0.038837</td>
<td>16</td>
<td>12</td>
<td>1.658869</td>
<td>0.469340</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>0.843910</td>
<td>0.025272</td>
<td>17</td>
<td>13</td>
<td>1.688552</td>
<td>0.440820</td>
</tr>
<tr>
<td>9</td>
<td>500</td>
<td>0.223061</td>
<td>0.004876</td>
<td>18</td>
<td>14</td>
<td>1.842303</td>
<td>0.421160</td>
</tr>
</tbody>
</table>

Table 7: Relationship between diagonal dominance concept (DD) and CC (Zms) - fourth convolutional layer FCCM

Figure 8: Convolutional Complementary Credibility - Variance Scale Factor
databases defined in this study, taking as reference the OLE database. In this stage, the model was compared to two commonly used models for estimating the credibility: the Buhlmann-Straub theory (i.e. $BS - Zms$, $BS - OpV$) and the $FCM$ credibility model (i.e. $FCM - Zms$, $FCM - OpV$) [49]. Here it can be seen that the convolutional mechanism that supports the structure of ECFM managed better to attenuate the losses, as the magnitude of losses ($ML$) increased for the different risk scenarios. In general, the $MFC - DL$ reached $Zms$ values close to one more quickly than the other models, this fact shows that the model tends to favor internal losses OLE, maintaining at all times a balance with respect to the $l - ALEs$ losses, which evidences the capacity of the model against risk mitigation.

Figure 11 shows the behavior of $MFC - DL$ against the estimation of the $OpVar$ value integrating different $ALEs$ for two groups of risks (i.e. Scenarios $ML \leq 1.0$, Scenarios $ML > 11.0$). Figure 11 (a) illustrates that $MFC - DL$ achieved the highest values of credibility ($Zms$); this is mainly due to the loss magnitude for risk scenarios with $ML \leq 1.0$ and it clearly shows that the $MFC - DL$ does not tend to have lower losses by integrating scenarios with lower losses. Figure 11 (b) illustrates that $MFC - DL$ behaves similarly to $BS - LC$ in relation to credibility for scenarios with $ML > 1.0$; however, $MFC - DL$ showed lower losses, which reinforces its capacity against risk mitigation based on the credibility concepts using multiple sources of risks (i.e. $l - ALEs$).

Figure 12 shows the $EFCM$ structure for the $11 - ALE$ ($ML = 7.834761$). Here, it can be observed that
the ZC credibility (i.e. Zms) increases, taking as reference the main diagonal (higher OLE losses - higher ALE losses); this is in accordance with the convolutional mechanism that integrates this structure. According to the losses located below the main diagonal (i.e. 11 – ALE losses zone), the MFC – DL favors credibility towards such losses as shown by the values of credibility ZC (Zms) in this zone. According to the losses located above the main diagonal (i.e OLE losses zone), the MFC – DL favors the credibility towards such losses. However, the structure of the EFCM shows in general the dominance of the OLE credibility over the 11 – ALE credibility, a fact that is corroborated by the behavior exhibited by the model in Figure 11 for ML > 1.0, which also shows the good risk mitigation that the MFC – DL generates, specially for l – ALEs with high loss events, and in general for the loss component (LC).

### 4.4. Fully connected layer setup

Table 9 and Figure 13 show the general behavior achieved by the FCL against the estimation of \( y_{\text{OpVar}} \) for each 1000 – ALEv reference scenarios after 1000 cycles of learning (MFC – DL – OpVar Scenario). The error metrics FB and MRE took negative values, which indicates that the MFC – DL tends to underestimate the \( y_{\text{OpVar,k}} \). However, the GM and the VG reached values close to one, which indicates that the LC distribution representing the \( y_{\text{OpVar,k}} \) and the LC distribution representing the FCL model (\( y_{\text{OpVar}} \)) have similar structures. The quality of the model in estimating \( y_{\text{OpVar,k}} \) is also reflected through the values obtained by the IOA (Figure 13) and the FAC2 index close to 90%. Also, the values achieved by the UAPC2 and the NMSE indexes were close to zero. This evidences in general the good behavior of the MFC – DL regarding the FCL configuration. According to the structure of the model, it is important to mention that the FCL supports the learning process with regards to the \( y_{\text{OpVar}} \) estimation, since the first layers are considered as layers of convolutional deep beliefs, which do not integrate learning mechanisms for adaption.

<table>
<thead>
<tr>
<th>FB</th>
<th>NMSE</th>
<th>GM</th>
<th>GV</th>
<th>FAC2</th>
<th>IOA</th>
<th>UAPC2</th>
<th>MRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.004079</td>
<td>0.000365</td>
<td>0.995733</td>
<td>1.000358</td>
<td>1</td>
<td>0.88627</td>
<td>0.077001</td>
<td>-0.004457</td>
</tr>
</tbody>
</table>

Table 9: Statistical Index – FCL Behavior

Table 10 and Figure 14 show the behavior of the model against a configuration scenario for adaption and learning versus estimation of \( y_{\text{OpVar,k}} \) (MFC – DL – LC), and three risk scenarios (i.e. MFC – DL(ML ≤ 1.0), MFC – DL(ML ≈ 1.0), MFC – DL(ML > 1.0)) in absence of a learning mechanism. Figure 14 shows that the
Figure 11: Operational value at risk - (a) ALE Scenarios (ML ≤ 1.0) (b) ALE Scenarios (ML > 1.0)

Figure 12: Extended Fuzzy Credibility Map OLE Reference vs ALE 11 (ML=7.834761)

proposed MFC – DL achieved an OpVar value lower than the OpVar values estimated by the references scenarios: Buhlmann theory (Z – BS) and Fuzzy Cognitive Maps (Z – FCM). Regarding the scenarios in absence of a learning mechanism, the losses estimated by the MFC – DL were in agreement with the losses magnitude of each scenario. Table 10 and Figure 14 also show that the MFC – DL model yielded probability distributions with structures (slender and long tail distributions) similar to those recommended by the Basel II agreement for modeling this type of risk: Lognormal, Log-logistic, Weibull and Generalized Pareto, which also shows its structural stability against LC modeling. This stability concept is reinforced by the loss coverage indexes (CUL – CEL), which were in general above 90% on average, where the LC distributions maintains the same ratio between EL and UL losses.

Table 10 and Figure 15 also show the sensitivity of the model against the estimation of LC for the three risk scenarios in the absence of a learning mechanism. It is highlighted that the LC estimated by the MFC – DL grouped lower losses for the group of scenarios with lower loss magnitudes (MFC – DL(ML ≤ 1.0)). These losses for LC were much higher for the group of scenarios with much higher loss magnitudes (MFC – DL(ML > 1.0)). Regarding central losses (MFC – DL – LC(ML ≈ 1.0)), the performance of the MFC – DL was located between the two previous scenarios as evidenced by the OpVar value. The evolution of the OpVar was mainly promoted by the SI that define the LC, which increased, as the losses were much lower (Dimensional stability). This fact clearly evidence the flexibility and dimensional stability reached by the model in the estimation of losses with different magnitudes, configuring the MFC – DL as a tool for comprehensive operational risk management using information internal and external to an organization or financial entity.
Table 10: Loss Component – ALD Distributions

<table>
<thead>
<tr>
<th>Distribution 1</th>
<th>BS-LC</th>
<th>FCM-LC</th>
<th>MFC-DL-LC</th>
<th>MFC-DL$(ML &lt; 1.0)$</th>
<th>MFC-DL$(ML \approx 1.0)$</th>
<th>MFC-DL$(ML &gt; 1.0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.LogL</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>3.175E+03</td>
<td>2.941E+03</td>
<td>2.534E+03</td>
<td>1.8352E+03</td>
<td>2.3112E+03</td>
<td>2.8724E+03</td>
</tr>
<tr>
<td>Distribution 2</td>
<td>G.Pareto</td>
<td>G.Pareto</td>
<td>G.Pareto</td>
<td>Log-Logistic</td>
<td>G.Pareto</td>
<td>Log-Logistic</td>
</tr>
<tr>
<td>N.LogL</td>
<td>3.172E+03</td>
<td>2.9466E+03</td>
<td>2.5313E+03</td>
<td>1.8455E+03</td>
<td>2.3092E+03</td>
<td>2.8802E+03</td>
</tr>
<tr>
<td>Distribution 3</td>
<td>Log-Logistic</td>
<td>Weibull</td>
<td>Log-Logistic</td>
<td>G.Pareto</td>
<td>Log-Logistic</td>
<td>G.Pareto</td>
</tr>
<tr>
<td>N.LogL</td>
<td>3.186E+03</td>
<td>3.0274E+03</td>
<td>2.5414E+03</td>
<td>1.881E+03</td>
<td>2.3173E+03</td>
<td>2.9006E+03</td>
</tr>
<tr>
<td>Expected Losses (EL)</td>
<td>9.55671</td>
<td>11.21249</td>
<td>5.18361</td>
<td>2.43748</td>
<td>4.21816</td>
<td>7.30411</td>
</tr>
<tr>
<td>Unexpected Losses (UL)</td>
<td>3.59815</td>
<td>5.97444</td>
<td>2.51515</td>
<td>1.10705</td>
<td>2.31097</td>
<td>3.5423</td>
</tr>
<tr>
<td>OpVar (99.9%)</td>
<td>173.13461</td>
<td>104.21755</td>
<td>71.11721</td>
<td>32.88721</td>
<td>44.77944</td>
<td>95.76474</td>
</tr>
<tr>
<td>NEL</td>
<td>720</td>
<td>665</td>
<td>694</td>
<td>684</td>
<td>673</td>
<td>688</td>
</tr>
<tr>
<td>NUL</td>
<td>279</td>
<td>334</td>
<td>305</td>
<td>315</td>
<td>326</td>
<td>311</td>
</tr>
<tr>
<td>NSL</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CEL</td>
<td>0.94481</td>
<td>0.89241</td>
<td>0.92711</td>
<td>0.92588</td>
<td>0.90582</td>
<td>0.92373</td>
</tr>
<tr>
<td>CUL</td>
<td>0.97922</td>
<td>0.94267</td>
<td>0.96468</td>
<td>0.96634</td>
<td>0.94839</td>
<td>0.96301</td>
</tr>
</tbody>
</table>

Figure 14: Loss Component (LC) – Different models

Figure 13: Index of Agreement (IOA) – Learning behavior with regard the ALEvs.
5. Conclusions and future work

The proposed MFC–DL model allows the OR modeling in a comprehensive way, integrating different databases of loss events (l–ALE i.e. risk scenario) for the estimation of ORC (i.e. OpVar). The model allowed the identification of credibility features in l–ALE databases, integrating a Fuzzy Cognitive Map (FCM) for joint representation of RVs as LRVs, as well as a Fuzzy Credibility Compact Map (FCCM) to create fuzzy credibility patterns for the estimation of LC from multi-sources of loss events or risk scenarios. In general, the MFC – DL managed to overcome the limitations imposed by the qualitative uncertain low frequency with which a loss event is generated within an organization or financial entity, configuring the concept of fuzzy multidimensional credibility.

The flexibility and autonomy were evidenced through the structural and dimensional stability reached by the model in the estimation of LC integrating different l–ALE scenarios with different magnitudes of losses. Here, the structural stability was determined by the structure of the probability distributions given by the MFC–DL for modeling the LC, which were in agreement with the probability distributions suggested by the Basel II agreement for the modeling of this risk. This fact shows the ability of the MFC–DL to identify the structure of losses in a specific risk scenario for adaption and learning. Meanwhile, the dimensional stability was evidenced through the MFC–DL sensitivity in the configuration of its structure for scenarios with different losses magnitude.

In general, the MFC–DL model reached much higher credibility values in a faster way than the reference models used for this study (i.e. BS–LC, FMC–LC), as the losses magnitude increased for different risk scenarios. It is also important to mention that these credibility values were also increasing as the losses magnitude for a risk scenario were decreasing. This fact was mainly due to the distance generated by the convolutional mechanisms between the fuzzy sets that define a risk scenario and fuzzy sets that make up the OLE reference database regarding the estimation of VHM and EPV values. This determines the granularity of the model versus the OpVar estimation.

The proposed model can be considered an integral tool for estimating the OpVar value, taking into account three perspectives: (1) the regulators (Basel Accords Committee) that have the possibility to integrate the experts’ criterion into operational risk modeling; (2) the insurers that can integrate it into a single risk event operational model from different sources of losses, which are, from an insurers perspective, subject of coverage; (3) organizations and financial entities can use it as a tool to evaluate a priori the OR associated with the operations of new financial products, technological platforms or electronic channels.

It is noticed that the flexibility achieved by the proposed model allowed to obtain much higher credibility values than those reported in the recent literature. Unlike other available models for modeling operational risk, we achieve a better characterization of losses (internal and external databases) by considering linguistic random variables. It is also important to highlight that the convolutional mechanisms of the model made it less sensitive to the presence of extreme losses due to its response to the intrinsic structure of losses. In addition to the above, the proposed model is in accordance with all the guidelines established by the Basel agreements, which makes it a reference model for estimating ORC, and in general for modeling OR.
For the forecasting and management of OR in real time, the authors consider important the creation of convolutional mechanisms that allow the generation of forecast maps to estimate the credibility ($Z_{ms}$), taking into account the dynamic generation of new loss events. Here, the convolutional mechanism falls on a $l - ALE$ that is receiving new loss events in real time. In this context, the Estimation Distribution Algorithms (EDA’s) [31] and Lagrangian tracking models become potential alternative to identify evolutionary patterns for fuzzy sets that make up the aforementioned $l - ALE$, leading from the FCCM concept to the Evolutionary Fuzzy Credibility Compact Maps (EFCCM) concept in the estimation of credibility [52].

References


