New Consistency Properties for Preference Relations

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Abstract

Consistency of preference relations is associated with the study of the transitivity property. This is clearly related to the scales used for associating preferences values to judgements, and the conditions used to represent transitivity. Both issues, the scale to use and the consistency property, have been studied in the multiplicative model as well as in the fuzzy one. However, we make note the existence of a conflict between the consistency property for multiplicative preference relations and the scale proposed to provide a such preference relation. Obviously, the same problem exists when dealing with fuzzy preference relations. In order to overcome this problem, in this paper we analyze the properties to be verified by a function, \( T \), in order to obtain the value of preference of the alternative \( x_i \) over the alternative \( x_k \) when we already have the values of the preference of \( x_i \) over \( x_j \), and of \( x_j \) over \( x_k \). As a consequence, we define \( T \)-additive transitivity property as the consistency property for fuzzy preference relations and \( T \)-multiplicative transitivity property for the case of multiplicative preference relations.

Keywords: Decision making; Consistency; Transitivity; Fuzzy preference relations; Multiplicative preference relations.
1. Introduction

It is widely acknowledged that fuzzy sets play an important role in modelling decision processes because human judgements, including preferences, are often vague. We should consider, for instance, the situation when a set of feasible options have to be pairwise compared. In this case, the opinions of the experts are usually described using preference relations. Many important decision models have been developed using mainly two kinds of preference relations: fuzzy preference relations [1–3,7,16] and multiplicative preference relations [2,5,6,11–13].

In order to make consistent choices when dealing with preference relations a set of properties or restrictions to be satisfied by such preference relations have been suggested. In the multiplicative model, a multiplicative preference relation is consistent when it verifies the so called multiplicative consistency property [11]. The results obtained in [2] imply that a fuzzy preference relation is consistent if and only if the corresponding multiplicative preference relation is consistent. Therefore, a fuzzy preference relation is considered consistent when it verifies the so-called additive consistency property.

However, there exists a conflict between the multiplicative and additive consistency properties and the scales used to assign preference values to judgements. There exist many arguments to support that a change in the scales used in the multiplicative and fuzzy models seems unpractical and unrealistic. Therefore, the only possible solution to overcome the existing conflict seems to be a change in the definition of the consistency properties. This will be analyzed in more depth in the following section.

In this paper we address this problem and study the properties for a fuzzy reciprocal preference relation to be considered a consistent one. We will study the general conditions of a function $T : [0,1] \times [0,1] \rightarrow [0,1]$ to obtain $p_{ik}$ from the pair of values $(p_{ij}, p_{jk})$. Using this function, we introduce the concepts of the $T$-additive consistency property in the case of fuzzy preference relations, which consist of a relaxation of the additive consistency property.

In order to do this, the rest of the paper is organized as follows. In section 2, we present an overview of the consistency properties defined for preference relations. In section 3, we show the existence of a conflict between the consistency properties and the scales used to provide preference relations. This conflict means that a modification of the actual consistency properties for both multiplicative and fuzzy preference relations is needed. In section 4, we study properties to be verified by a function, $T$, in order to obtain the preference value $p_{ik}$ when the pair of preference values $(p_{ij}, p_{jk})$ is known. As a result of this, in section 4 the concepts of $T$-additive and $T$-multiplicative transitivity properties are introduced. Finally, in section 6 we draw our conclusions.
2. Consistency Properties and Preference Relations

In a preference relation an expert associates to each pair of alternatives a real number that reflects the preference degree, or the ratio of preference intensity, of the first alternative over, or to that of, the second one. Two questions immediately arise when doing this:

- Which scale should be used to associate preference values to judgements?
- Which conditions have to be satisfied in order to obtain consistent results?

The answer to the first question depends on the selection model we are working with. The most well-known selection models are:

(i) Fuzzy model. In this case, preferences are represented by a fuzzy preference relation $P$ on a set of alternatives $X$, i.e. a fuzzy set on the product set $X \times X$, which is characterized by its membership function $\mu_P : X \times X \rightarrow [0, 1]$ [17]. This implies that the scale to use in the fuzzy model is the closed interval $[0, 1]$.

(ii) Multiplicative model. In this case, preferences are represented using a multiplicative preference relation, $A = (a_{ij})$, on a set of alternatives $X$, being $a_{ij}$ interpreted as the ratio of the preference intensity of alternative $x_i$ to that of $x_j$. According to Miller’s study [10], Saaty suggests measuring $a_{ij}$ using as ratio scale, and precisely the $1/9$ scale [11], or more generally the closed interval $[1/9, 9]$.

With respect to the second question, we agree with Saaty [11] in the sense that lack of consistency in decision making can lead to inconsistent conclusions; that is why it is important to study conditions under which consistency is satisfied.

In a crisp context, where an expert provides his/her opinion on the set of alternatives, $X = \{x_1, x_2, \ldots, x_n; n \geq 2\}$, by means of a binary preference relation, $R$, the concept of consistency has traditionally been defined in terms of acyclicity [14], that is the absence of sequences such as $x_1, x_2, \ldots, x_k(x_{k+1} = x_1)$ with $x_jRx_{j+1} \forall j = 1, \ldots, k$.

In a fuzzy context, a traditional requirement to characterize consistency is using transitivity, in the sense that if an alternative $x_i$ is preferred to alternative $x_j$ and this one to $x_k$ then alternative $x_i$ should be preferred to $x_k$, although stronger conditions have been given to define consistency [9,15–17].

In the multiplicative model, what Saaty means by consistency is what he calls cardinal transitivity in the strength of preferences, which is a stronger condition than the traditional requirement of the transitivity of preferences. Thereby, the definition of consistency proposed by Saaty is the following [11]

**Definition 1.** A reciprocal multiplicative preference relation $A = (a_{ij})$ is consistent if

$$a_{ij} \cdot a_{jk} = a_{ik} \ \forall i, j, k = 1, \ldots, n.$$ 

Inconsistency for Saaty is a violation of proportionality which may not entail violation of transitivity [11].
Some of the suggested properties in the case of fuzzy preference relations are:

(i) Triangle condition [9]: $p_{ij} + p_{jk} \geq p_{ik} \ \forall i, j, k$

(ii) Weak transitivity [16]: $\forall i, j, k: \min\{p_{ij}, p_{jk}\} \geq 0.5 \Rightarrow p_{ik} \geq 0.5$

(iii) Max-min transitivity [4,17]: $p_{ik} \geq \min\{p_{ij}, p_{jk}\} \ \forall i, j, k$

(iv) Max-max transitivity [4,17]: $p_{ik} \geq \max\{p_{ij}, p_{jk}\} \ \forall i, j, k$

(v) Restricted Max-min transitivity [16]: $\forall i, j, k: \min\{p_{ij}, p_{jk}\} \geq 0.5 \Rightarrow p_{ik} \geq \min\{p_{ij}, p_{jk}\}$

(vi) Restricted Max-max transitivity [16]: $\forall i, j, k: \min\{p_{ij}, p_{jk}\} \geq 0.5 \Rightarrow p_{ik} \geq \max\{p_{ij}, p_{jk}\}$

(vii) Multiplicative transitivity [16]: $\frac{p_{ik}}{p_{ij}} \cdot \frac{p_{ik}}{p_{jk}} = \frac{p_{ik}}{p_{ij}} \ \forall i, j, k$

(viii) Additive transitivity [15,16]: $(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \ \forall i, j, k$ or equivalently $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \ \forall i, j, k$

In [2] we obtained the transformation function between multiplicative and fuzzy preference relations, which is given in the following result:

**Proposition 1.** Suppose that we have a set of alternatives, $X = \{x_1, \ldots, x_n\}$, and associated with it a multiplicative reciprocal preference relation $A = (a_{ij})$, with $a_{ij} \in [1/9, 9]$ and $a_{ij} \cdot a_{ji} = 1, \forall i, j$. Then the corresponding fuzzy reciprocal preference relation, $P = (p_{ij})$, associated with $A$, with $p_{ij} \in [0, 1]$ and $p_{ij} + p_{ji} = 1, \forall i, j$, is given as follows:

$$p_{ij} = f(a_{ij}) = \frac{1}{2} \left(1 + \log_9 a_{ij}\right).$$

The above transformation function is bijective and, therefore, allows us to transpose concepts that have been defined for fuzzy preference relations to multiplicative preference relations. Indeed, applying the above function we show that additive transitivity property for fuzzy preference relation can be seen as the parallel concept of Saaty’s multiplicative consistency property:

**Proposition 2.** Suppose that $A = (a_{ij})$ is a multiplicative consistent preference relation. Then, the corresponding reciprocal fuzzy preference relation, $P = f(A)$, associated with $A$, being $p_{ij} = f(a_{ij}) = \frac{1}{2} \cdot (1 + \log_9 a_{ij})$ verifies additive transitivity property.

All this lead us to define the concept of consistent fuzzy preference relation as in the following definition [8]:

**Definition 2.** A fuzzy reciprocal preference relation $P = (p_{ij})$ is consistent if

$$p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \ \forall i, j, k = 1, \ldots, n.$$

We will use the term **additive consistency property** to refer to this consistency property for fuzzy reciprocal preference relations. In the next section we study
3. Why modify the consistency properties?

The following simple example will show that there exists a conflict between the scales used to associate multiplicative preference values to judgements and the definition of consistency given by Saaty. Let us suppose a set of three alternatives \( \{x_1, x_2, x_3\} \) on which an expert provides the following judgements: alternative \( x_1 \) is considerably more important than alternative \( x_2 \) and this one demonstrably or overwhelming more important than alternative \( x_3 \). In such a case, using Saaty’s 1-9 scale, we would have the values \( a_{12} = 5 \) and \( a_{23} = 7 \).

On the one hand, if we want to maintain the multiplicative consistency property (see Appendix A.1) then, according to Saaty [11], we would have to assign the value \( a_{13} = a_{12} \cdot a_{23} = 35 \), and the only solution would be using the following consistent reciprocal multiplicative preference relation

\[
A = \begin{pmatrix}
1 & 5 & 35 \\
1/3 & 1 & 7 \\
1/35 & 1/5 & 1
\end{pmatrix}.
\]

Therefore, to avoid such a type of conflict we could proceed by choosing a different scale for providing judgements or by modifying the above definition. With respect to the first question, the use of any other scale of the form \([1/a, a]\), \(a \in \mathbb{R}^+\), would not make this conflict disappear, which means that the only possible solution to overcome this conflict would consist of using the scale of pairwise comparison from 0 to \( +\infty \). However, as Saaty points out in [11], this may not be useful at all because it assumes that the human judgement is capable of comparing the relative dominance of any two objects, which is not the case.

On the other hand, we note that if \( a_{13} \in [7, 9] \) transitivity still holds. We analyze this fact by means of the measure of consistency proposed by Saaty. In [11] Saaty shows that a reciprocal multiplicative preference relation is consistent if and only if its maximum or principal eigenvalue \( \lambda_{\text{max}} \) is equal to the number of alternatives \( n \). However, because perfect consistency is difficult to obtain in practice, especially when measuring preferences on a set with a large number of alternatives, Saaty defined a consistency index \( (CI = \lambda_{\text{max}} - n) \) that reflects the deviation from consistency of all the \( a_{ij} \) of a particular reciprocal multiplicative preference relation from the estimated ratio of priorities \( w_i/w_j \).

A measure of inconsistency independent of the order of the reciprocal multiplicative preference relation is defined as the consistency ratio \( (CR) \). This is obtained by taking the ratio of \( CI \) to the random index \( (RI) \), which is an average consistency index of a sample set of randomly generated reciprocal matrices from the scale 1 to 9 (size 500 up to 11 by 11 matrices, and size 100 for squares matrices of orders...
12, 13, 14 and 15). For this consistency measure, he proposed a threshold of 0.10 to accept the reciprocal multiplicative preference relation as consistent. When the CR is greater than 0.10 then, in order to improve consistency, those judgements with a greater difference $a_{ij}$ and $w_i/w_j$, are usually modified and a new priority vector is derived.

In our previous example we observe that the conflict between the multiplicative consistency property and the scale used by Saaty arises because if we impose consistency then we get values outside the range $[1/9, 9]$. If we restrict the possible values of $a_{13}$ to be in $[1/9, 9]$, then it is clear than in this case alternative $x_1$ should be considered as overwhelming more important than alternative $x_3$, and thus the value of $a_{13}$ should be greater or equal to 7. If $a_{13} = 7$ we get a C.R. value of 0.25412, with $a_{13} = 8$ a C.R. value of 0.212892 and with $a_{13} = 9$ a C.R. value of 0.179714, all of them greater than the minimum 0.10 for considering any reciprocal multiplicative preference relation consistent in this situation.

All these considerations mean that if we do not change the scale used to associate preference values to judgement or want to have a homogeneous scale when working in a group decision context, then the above definition 1 and definition 2 of consistency of preference relations should be modified.

Obviously, a similar analysis in the case of working with the fuzzy model can be carried out concluding that the same conflict also exists. In the next section, we will study the general conditions to be satisfied by a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ so that it can be used to obtain the preference value of the alternative $x_i$ over the alternative $x_k$, $p_{ik}$, from the preference values of $x_i$ over $x_j$ and of $x_j$ over $x_k$, $\{p_{ij}, p_{jk}\}$.

4. The value of $p_{ik}$ with known $(p_{ij}, p_{jk})$

If we do want to compare the alternatives $x_i$ and $x_k$, but cannot do it directly, and have an alternative $x_j$ of which we we know the exact values of $p_{ij}$ and $p_{jk}$, then we can establish a broad comparison between alternatives $x_i$ and $x_k$ on the basis of the values $p_{ij}$ and $p_{jk}$. Indeed, we can distinguish the following cases:

Case 1. $p_{ij} = 0.5 \ (p_{jk} = 0.5)$ which means that $x_i \sim x_j \ (x_j \sim x_k)$ and as a consequence the strength of preference between $x_i$ and $x_k$ should be the same as the one between $x_j$ and $x_k$. We then have: $p_{ik} = p_{jk} \ (p_{ik} = p_{jk})$.

Case 2. $p_{ij} > 0.5 \ and \ p_{jk} > 0.5$. In this case, alternative $x_i$ is preferred to alternative $x_j$ ($x_i \succ x_j$) and alternative $x_j$ is preferred to alternative $x_k$ ($x_j \succ x_k$). We then have that $x_i \succ x_j \succ x_k$ which implies $x_i \succ x_k$ and therefore $p_{ik} > 0.5$. Furthermore, in these cases restricted max-max transitivity should be imposed, which means that $x_i$ should be preferred to $x_k$ with a degree of intensity at least equal to the maximum of the intensities $p_{ij}$ and $p_{jk}$: $p_{ik} \geq \max\{p_{ij}, p_{jk}\}$, where the equality holds only when there exists indifference between at least one of the alternatives.
and \( x_j \), i.e., \( p_{ij} = 0.5 \) or \( p_{jk} = 0.5 \), as we have said in case 1. As a result, in this case \( p_{ik} > \max\{p_{ij}, p_{jk}\} \) should be verified.

**Case 3.** \( p_{ij} > 0.5 \) and \( p_{jk} < 0.5 \) which is equivalent to \( p_{ij} > 0.5 \) and \( p_{kj} = 1 - p_{jk} > 0.5 \), that is: \( x_i > x_j \) and \( x_k > x_j \). The comparison of alternatives \( x_i \) and \( x_j \) is done by comparing the intensities of preferences of them over the alternative \( x_j \). An indifference situation between \( x_i \) and \( x_k \) would exist only when both alternatives are preferred over \( x_j \) with the same intensity, while the alternative with greater intensity of preference over \( x_j \) should be preferred to the other one. This is summarized in the following way:

\[
\begin{align*}
&\begin{cases}
  x_i \sim x_k & \text{if } p_{ij} = p_{kj} \iff p_{ij} + p_{jk} = 1 \\
  x_i > x_k & \text{if } p_{ij} > p_{kj} \iff p_{ij} + p_{jk} > 1 \\
  x_i < x_k & \text{if } p_{ij} < p_{kj} \iff p_{ij} + p_{jk} < 1
\end{cases} \\
\Rightarrow
\begin{cases}
  p_{ik} = 0.5 & \text{if } p_{ij} + p_{jk} = 1 \\
  p_{ik} > 0.5 & \text{if } p_{ij} + p_{jk} > 1 \\
  p_{ik} < 0.5 & \text{if } p_{ij} + p_{jk} < 1
\end{cases}
\end{align*}
\]

It is obvious that the greater the value \( |p_{ij} + p_{jk} - 1| \) the greater \( |p_{ik} - 0.5| \).

**Case 4.** \( p_{ij} < 0.5 \) and \( p_{jk} > 0.5 \). This case is analogous to case 3 and the same conclusion is obtained.

**Case 5.** \( p_{ij} < 0.5 \) and \( p_{jk} < 0.5 \). This case is analogous to case 2 and we obtain that \( p_{ik} < \min\{p_{ij}, p_{jk}\} \).

Cases 1 to 5 suggest that the value of \( p_{ik} - 0.5 \) is related to the value \( p_{ij} + p_{jk} - 1 = (p_{ij} - 0.5) + (p_{jk} - 0.5) \), and therefore there exists a function \( T : [0,1]^2 \rightarrow [0,1] \), such that

\[ p_{ik} = T(p_{ij}, p_{jk}) \]

The above cases mean that function \( T \) verifies:

(i) \( T(0.5, y) = y \ \forall y \)

(ii) \( T \) is increasing in the interval \([0.5,1] \times [0.5,1]\) with respect to the value \( \max\{x,y\} \) and \( T(x,y) \geq \max\{x,y\} \) being equal only in the case \( \min\{x,y\} = 0.5 \).

(iii) \( T \) is increasing in the interval \([0,0.5] \times [0,0.5]\) with respect to the value \( \min\{x,y\} \) and \( T(x,y) \leq \min\{x,y\} \) being equal only in the case \( \max\{x,y\} = 0.5 \).

(iv) \( T \) is increasing in the sets \([0,0.5] \times (0.5,1] \) and \((0.5,1] \times [0,0.5]\) with respect to the value \( x + y - 1 \) and takes the value \( 0.5 \) when \( x + y - 1 = 0 \).

(v) \( T(1-x, 1-y) = 1 - T(x,y) \) is equivalent to \( T(x,y) = T(y,x) \), i.e. \( T \) is a symmetric function.

Another desirable property to be verified by function \( T \) should be that of continuity as it is expected that a slight change of the values in \( (p_{ij}, p_{jk}) \) should produce a slight change in the value \( p_{ik} \).
In order to know more about possible function \( T \) satisfying the above conditions, we may start assuming that \( T(x, y) = f(x + y) \), with \( f : [0, 2] \to [0, 1] \) a continuous and increasing such that \( f(1) = 0.5 \). The linear solutions satisfying this last properties take the form \( f_1(z) = z/2 \) and \( f_2(z) = z - 0.5 \).

The first linear solution gives \( p_{ik} = T(p_{ij}, p_{jk}) = (p_{ij} + p_{jk})/2 \), which fails to satisfy property restricted max-max transitivity, because \( \min\{p_{ij}, p_{jk}\} \leq (p_{ij} + p_{jk})/2 \leq \max\{p_{ij}, p_{jk}\} \) and in the case of \( p_{ij} \to 0.5 \land p_{jk} \to 1 \) \( (p_{ij} \to 1 \land p_{jk} \to 0.5) \) we get that \( p_{ik} \to 0.75 \) instead of \( p_{ik} \to p_{jk} = 1 \) \( (p_{ik} \to p_{ij} = 1) \).

With the second linear solution we get \( p_{ik} = T(p_{ij}, p_{jk}) = p_{ij} + p_{jk} - 0.5 \), which coincides with additive transitivity. In this case \( T \) verifies restricted max-max transitivity but fails to satisfy \( p_{ij} \to 0 \land p_{jk} \to 0 \) \( (p_{ij} \to 1 \land p_{jk} \to 1) \implies p_{ik} \to 0 \) \( (p_{ik} \to 1) \).

In fact, if \( x, y \leq 0.5 \) and \( x + y \leq 0.5 \) then the above function gives negative values of \( p_{ik} \), while in the case of \( x, y \geq 0.5 \) and \( x + y \geq 1.5 \) we obtain values of \( p_{ik} \) greater than 1. We can overcome this problem by defining \( T \) as a piecewise function. A possible modification of function \( T \) would be the following:

\[
T(x, y) = \begin{cases} 
\min\{x, y\} & x + y \leq 0.5 \\
\max\{x, y\} & x + y \geq 1.5 \\
x + y - 0.5 & \text{otherwise}
\end{cases}
\]

This function has a drawback in that it treats an infinite number of different cases as equal. For example when \( p_{ij} = 0.9 \) and \( p_{jk} \in [0.6, 0.9] \) this function returns the value \( p_{ij} = 0.9 \) and it is obvious that \( x_i \) should be preferred to \( x_k \) with a degree of intensity in the case of \( p_{jk} = 0.9 \) greater than when \( p_{jk} = 0.6 \). Furthermore, this function it is not continuous because

\[
\lim_{x+y \to 1.5^-} T(x, y) = 1
\]

and

\[
\lim_{x+y \to 1.5^+} T(x, y) = \max\{x, y\},
\]

being these limits the same only when \( x \to 1 \) or \( y \to 1 \). The same can be said when \( x + y \to 0.5 \).

5. \( T \)-additive transitivity and \( T \)-multiplicative transitivity

If we want function \( T \) to be continuous, then we have to narrow the application of additive transitivity to the case of \( (p_{ij}, p_{jk}) \in [0, 0.5] \times (0.5, 1] \cup (0.5, 1] \times [0, 0.5] \) where it shows good behaviour, that is:

\[
T(x, y) = \begin{cases} 
\min\{x, y\} & x, y \in [0, 0.5] \\
\max\{x, y\} & x, y \in [0.5, 1] \\
x + y - 0.5 & \text{otherwise}
\end{cases}
\]
This last expression of function $T$, although continuous, presents the same behaviour as the previous one in the sense of treating an infinite number of different cases as equal. Again, the same value is obtained for the two different pairs of values $(0.6, 0.9)$ and $(0.9, 0.9)$. So, some kind of strength with respect to the maximum (minimum) has to be incorporated in function $T$ to differentiate those tuples of values with the same minimum (maximum) value.

As a consequence, a better expression of such a function $T$ would be the following one:

$$T(x, y) = \begin{cases} 
\min\{x, y\} - h_1(x, y) & x, y \in [0, 0.5] \\
\max\{x, y\} + h_2(x, y) & x, y \in [0.5, 1] \\
x + y - 0.5 & \text{otherwise}
\end{cases}$$

where

$$h_1 : [0, 0.5]^2 \rightarrow [0, \min\{x, y\}] \subseteq [0, 0.5]$$

and

$$h_2 : [0.5, 1]^2 \rightarrow [0, 1 - \max\{x, y\}] \subseteq [0, 0.5]$$

are continuous and increasing functions satisfying

$$\min\{x, y\} \rightarrow 0.5 \Rightarrow h_1(x, y) \rightarrow 0$$

and

$$\max\{x, y\} \rightarrow 0.5 \Rightarrow h_2(x, y) \rightarrow 0$$

respectively.

All these considerations allow us to define a new transitivity condition, that we name $T$-additive transitivity:

**Definition 3.** A fuzzy preference relation $P$ is $T$-additive transitive if $p_{ik} = T(p_{ij}, p_{jk})$ being $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ a function verifying:

(i) $T(x, y) \geq \max\{x, y\} \ \forall x, y \in [0.5, 1]$

(ii) $T(x, y) \leq \min\{x, y\} \ \forall x, y \in [0, 0.5]$

(iii) $T(x, y) = x + y - 0.5 \ \text{otherwise}$

It is obvious that $T$-additive transitivity implies restricted max-max transitivity:

**Proposition 3.** $T$-additive transitivity implies restricted max-max transitivity, restricted max-min transitivity and weak transitivity.
Definition 4. A multiplicative preference relation $A = (a_{ij})$ is $T$-multiplicative transitive if the fuzzy preference relation $P = (p_{ij})$ is $T$-additive transitive, with $p_{ij} = f(a_{ij}) = \frac{1}{2} (1 + \log_9 a_{ij})$.

This definition implies:

Definition 5. A multiplicative preference relation $A$ is $T$-multiplicative transitive if $a_{ik} = T(a_{ij}, a_{jk})$ being $T : [1/9, 9] \times [1/9, 9] \rightarrow [1/9, 9]$ a function verifying:

(i) $T(x,y) \geq \max\{x,y\} \forall x,y \in [1,9]$

(ii) $T(x,y) \leq \min\{x,y\} \forall x,y \in [1/9,1]$

(iii) $T(x,y) = xy$ otherwise

6. Conclusions

In order to make consistent choices when dealing with preference relations a set of properties or restrictions to be satisfied by such preference relations have been suggested. In the multiplicative model, a multiplicative preference relation is consistent when it verifies the so called multiplicative consistency property. In the additive model, a fuzzy preference relation is considered consistent when it verifies the so called additive consistency property.

However, there exists a conflict between the multiplicative and additive consistency properties and the scales used to assign preference values to judgements. There exist many arguments to support that a change in the scales used in the multiplicative and fuzzy models seems unpractical and unrealistic. Therefore, the only possible solution to overcome the existing conflict seems to be a change of the definition of the consistency properties.

In this paper we have addressed this problem and have studied the properties to be verified for a preference relation to be considered a consistent one. We have introduced the concepts of $T$-additive consistency property in the case of fuzzy preference relations and $T$-multiplicative consistency property for multiplicative preference relations which consist of a relaxation of the additive and multiplicative consistency properties.

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